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Company valuation as result of risk analysis: replication approach as an alternative to the CAPM

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Market imperfections call into question the suitability of the CAPM for deriving the cost of capital. The valuation by incomplete replication introduces a valuation concept that takes capital market imperfections into account and derives the risk-adjusted cost of capital (or risk discounts) on the basis of corporate or investment planning and risk analysis. The risk measure is derived consistently (using risk analysis and Monte Carlo simulation) from the cash flows to be valued, that is, the earning risk. Historical stock returns of the valuation object are therefore not necessary. It can be shown that the valuation result of the CAPM can be derived using the approach of imperfect replication as a special case for perfect capital markets.

1. Introduction and overview

The idea of a capital market-oriented[1] company valuation has to be questioned due to many imperfections[2] of the capital market[3]. Especially the CAPM does not meet the challenges of a company valuation on imperfect capital markets due to the assumption of perfect and complete capital markets[4]. An improvement in the valuation results (e.g. as a basis for decision-making on the purchase of companies) if company-related factors (such as growth[5], return on equity or company-specific risks) are taken into account in the valuation models[6]. This leads to a replacement of capital market-oriented valuation approaches with procedures that - in the tradition of investment theoretical valuation approaches[7] - deal with earnings risks and not primarily with share price fluctuations; these are the semi-investment theoretical valuation methods described in this article, which are based on an analysis of business risks and the method of imperfect replication. The central advantage of the valuation approach is that it is based on only two low restrictive assumptions and, in particular, does not make any restrictive assumptions about the characteristics of the valuation subject or the capital market. In particular, there is no need to assume a perfect or complete or arbitrage-free capital market (the central assumption is simply the following: two payments at the same time have the same value if they match the expected value and the risk measure chosen by the valuation subject). Also rating and financing restrictions and insolvency costs are possible. Overall, an "idealized market calculus", as explained by *Ballwieser* (2010), is therefore not required for deriving the valuation equations[8]. So the actual approach presents an alternative to CAPM and implied cost of capital (*Bini*, 2018) to derive cost of capital. But it is not necessary to assume that the value is the market price. The valuation always takes place consistently from the perspective of the respective valuation subject (so that, for example, the degree of diversification of its assets achieved by this valuation subject always prevails, and not the diversification possibilities of other valuation subjects on the capital market).

In contrast to pure investment theoretical methods[9], the valuation (and the derivation of the cost of capital) takes place without the necessity of considering and simultaneously optimizing all investment and financing possibilities of the valuation subject (espe-

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[1] I.e. financial theoretical.

[2] See the overview of empirical studies at *Gleißner*, 2014.

[3] *Hering* 2014.

[4] See *Bini*, 2018, pp. 8-11; *Dempsey* 2013a and b; *Fernandez* 2013 and 2017; *Rossi*, 2016; the empirical studies (and alternative models) at *Fama/French*, 2015; *Blitz/Hanauer/Vidojevic/van Vliet*, 2018; *Kaserer/Hanauer*, 2017; *Ang et al* 2006 and 2009; *De Bondt/Thaler*, 1985 and 1987; *Jegadeesh/Titman*, 2011.

[5] Esp. of assets (see *Chen/Novy-Marx/Zhang*, 2011 and *Fama/French*, 2015.

[6] See *Ang et al* 2006 and 2009; the empirical studies (and alternative models) at *Fama/French*, 2015; *Kaserer/Hanauer*, 2017; *Walkshäusl*, 2013; Zhang, 2009.

[7] See *Matschke/Brösel*, 2013 and *Hering*, 2014.

[8] The explained semi-investment-theoretical valuation approach is new due to the lack of need for an idealized capital market or the existence of a utility function. It is not included in the survey of *Ballwieser*, 2010.

[9] See i.e.*Hering*, 2014; *Matschke/Brösel*, 2013; *Toll/Kintzel*, 2018.

cially by means of a simplex algorithm)[10]. In the tradition of risk-value models[11], valuation is performed by comparing the expected value of cash flows and their risks, expressed by a selected risk measure, with the risk-return profile of alternative investment opportunities (e.g. government bonds and equity indices available on the capital market). In accordance with the idea of "imperfect replication", the risky cash flow to be valued is thus expressed only in terms of the expected value and risk measure (R). It is only necessary to know the relevant information about two alternative investment opportunities (and not about the whole investment program)[12]. Accordingly, the valuation is based on a (μ,R)-preference function that includes the well-known (μ,s)-preference function of the Capital Asset Pricing Model (CAPM) as a special case. In contrast to the utility theoretical evaluation, knowledge of utility functions is also not required[13]. The great advantage of the valuation approach is that no (historical) capital market information about a company to be valued is required and the derivation of the cost of capital and company value from the analysis of the opportunities and risks of the company is possible. Risk analysis and Monte Carlo simulation for the aggregation of individual risks with reference to corporate planning provide the valuation-relevant information. Due to the consistent reference to the future and the consideration of future risks, the valuation approach outlined in this article is suitable for valuing existing options for action in the preparation of business decisions (e.g. in the context of a strategy assessment). This also explains the great importance of the valuation approaches presented here for financial corporate management (controlling). The central business task is a well-founded weighing of expected returns and risks in important decisions. The preparation of business decisions requires a well-founded strategy, operational planning based on it, an analysis of opportunities and threats and a risk-adequate evaluation of the options for action.

In this article we first discuss the challenges of a modern company valuation. We then analyse how a risk adjustment is made using the risk premium method and the certainty equivalence method. We then apply the certainty equivalence method to the CAPM. In the next section, we derive the valuation equation and the cost of capital using incomplete replication as an alternative to CAPM. Insolvency risk and rating are also taken into account.

The article is structured as follows. Section 2 addresses some of the key challenges of adequately capturing risks in the valuation of companies, such as the fact that business risks generally affect (1) the expected value of cash flows and (2) the cost of capital. Chapter 3 discusses the two ways in which business risk is accounted for in the valuation: the calculation of risk-adjusted cost of capital or of certainty equivalents. Section 4 shows how to derive valuation equations and cost of capital without assuming a perfect capital market (as in the case of the CAPM). In particular, the above-mentioned method makes it possible to derive risk-appropriate cost of capital directly from the results of the analysis of the company's risks. Special attention is also paid to the significance of the insolvency risk as well as the often existing rating and financing restrictions for the shareholder value (section 5). Section 6 explains the method by means of a simple case study before a short summary of the key statements.

2. Effects of risk on company value

When determining the value of a company as a future success value, it is necessary to observe certain equivalence principles[14]. It must be ensured that the "numerator" and the "denominator" of the valuation equation(s) are consistent with each other, especially with regard to risk assessment. This applies regardless of whether the risk adjustment of the cash flows is "in the denominator"[15] (in the case of the risk premium method) or "in the numerator" (in the case of the risk discount method)[16].

It should be noted that risks potentially affect (1) the expected value of the cash flow $E(\widetilde{CF})$ and (2) the cost of capital[17] at the same time. The effect of a risk R is simplistically shown in graph 1.

[10] The evaluation is understood as a comparison procedure and not as an optimization procedure.

[11] See *Sarin/Weber*, 1993.

[12] This is why the term "semi-investment theory approach" is also used here, see *Gleißner*, 2011.

[13] See *Bamberg/Dorfleitner/Krapp*, 2006 and the overview at *Schosser/Grottke*, 2013.

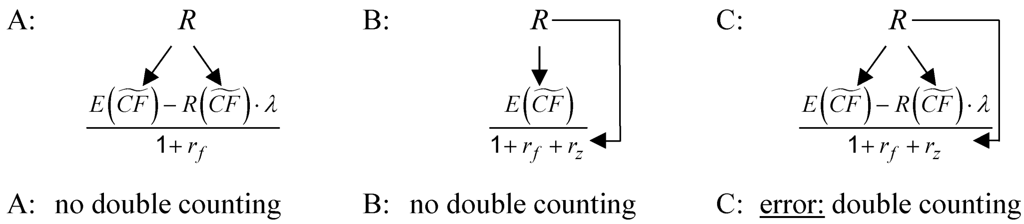
[14] See *Moxter*, 1983 and *Dehmel/Hommel*, 2017.

[15] Which is not recommended (see *Spremann*, 2004).

[16] See especially how to deal with insolvency risks that lead to a termination of the cash flow to the owner, *Gleißner*, 2017c.

[17] By means of a risk discount in the numerator or a risk premium r_z in the interest rate in the denominator.

Graph 1: Impact of risk of a company on its valuation components



This article deals with the methods of adequately recording risks in company valuation and shows in particular that the impact of risks on the expected value of cash flows (the numerator) and the discount rate (the denominator) can be derived consistently from a risk analysis of cash flows. An independent model for determining the discount rate - e.g. for deriving the CAPM beta based on fluctuations in equity returns or via a peer group - is not necessary[18]. The unrealistic assumptions of a perfect (or complete) capital market, as in financial theory valuation methods, are not required[19].

In addition, risks also affect the probability of default (the rating) and, via it, the level of cost of debt and the development over time of the expected value of the cash flows (a special case of the expected value effects explained above, see section 5).

In the following, "semi-investment" theoretical valuation methods are presented that take capital market imperfections into account and consistently calculate risk-adjusted cost of capital (or risk discounts) on the basis of corporate or investment planning. The information from risk analysis, financing restrictions, and insolvency risks are taken into account. The procedures can also be used if no capital market data is available for non-listed companies because the valuation is consistently derived from the uncertain cash flows themselves (business plan).

3. Fundamentals of risk adjustment in the evaluation of series of cash flows

The company valuation is based on the discounted cash flow method (DCF). Under the DCF method, the value of a company is determined on the basis of expected future cash flows[20]. These expected cash flows are derived from an integrated planning calculation. To determine the company value, the cash flows are discounted to the valuation date using a suitable capitalization interest rate (cost of capital).

The risk of future cash flows ($\widetilde{CF}$), i.e. the extent of possible deviations from the expected value ($E(\widetilde{CF})$) can be considered in the following two ways

- using the risk premium method, i.e. the calculation of the cost of capital[21]
- or using the certainty equivalence method (risk discount variant).

3.1 Cost of capital: the risk premium method

With the risk premium method, a risk premium (r_{CF}) is added to the risk-free interest rate (r_f). This results in a discount rate (c) (approximately the cost of capital) for discounting the expected future cash flows[22]. The formula for the discount rate is as follows:

$$c = r_f + r_{CF} \tag{1}$$

r_{CF} is usually determined as a function of equity yield risks, e.g. expressed by the beta factor of the CAPM. The extent to which this reflects the actual risks of the company, e.g. the volatility of cash flows $\sigma(\widetilde{CF})$ is, however, open. And only under specific additional assumptions, especially with regard to perfect capital market, the risks of the cash flows of the company are adequately recorded in r_{CF}.

The value of a risky cash flow ($\widetilde{CF}_1$) at time t = 0 is obtained by discounting the expected value $E(\widetilde{CF}_1)$ with the cost of capital c:

$$Value(\widetilde{CF}_1) = \frac{E(\widetilde{CF}_1)}{1+r_f+r_{CF}} \tag{2}$$

The risk premium method is often used in company valuation practice. However, it leads to valuation errors when a uniform risk premium is applied to both positive and negative cash flows[23]. This can be explained as follows. The basic idea behind discounting

[18] And generally not consistent with the valuation-relevant risk scope of the cash flow.

[19] See *Ballwieser*, 2008; see also the criticism at *Dempsey*, 2013; *Gleißner*, 2014; *Hering*, 2014 and *Fernández*, 2013 und 2017.

[20] I.e. free cash flows or flows to equity.

[21] This approach is usually applied in business valuation practice.

[22] But it is necessary to know the market price and to assume that the value is the price (see *Black*, 1986 and *Shleifer/Vishny*, 1992 und 1997 for some problems with this assumption).

[23] See *Spremann*, 2004, p. 253.

uncertain cash flows is that due to risk aversion, uncertain cash flows are assigned a lower value by discounting than certain cash flows. However, this is precisely not achieved by discounting negative cash flows: discounting negative cash flows increases the value because it becomes less negative[24]. It is therefore advisable to use the certainty equivalence method, which provides correct valuations[25].

3.2 Certainty equivalence method

The certainty equivalence method is based on the following equation:

$$(3) \qquad Value(\widetilde{CF}_1) = \frac{CE(\widetilde{CF}_1)}{1+r_f} = \frac{E(\widetilde{CF}_1) - \lambda_{CE} \cdot R(\widetilde{CF}_1)}{1+r_f}$$

λ_{CE} stands for the "market price of risk". This term expresses what additional return per unit for additionally accepted risk (measured in the selected risk measure $R(\widetilde{CF})$[26] for the alternative investment opportunity under consideration, e.g. the capital market) is to be expected. The scope of risk of a cash flow is recorded with a deduction in the numerator. A clear distinction is made between risk preference in the numerator and time preference (risk-free interest rate) in the denominator (*Ballwieser*, 1981).

The risk analysis of the cash flows to be valued leads to risk-adjusted risk measures that are not derived from historical stock returns. Suitable risk measures, such as value-at-risk, can take into account not only the standard deviation used in the beta factor but also the skewness and kurtosis of the distribution or a larger data density.

3.3 CAPM based on certainty equivalent and risk analysis as a special case

Even if an appraiser wishes to follow the traditional CAPM valuation approach, he or she should aggregate the valuation-relevant information on the risks of uncertain cash flows $\widetilde{CF}$ to an appropriate risk measure. This is made possible by the "risk discount variant" of CAPM, whose risk measure is based on the correlation between future cash flows and the market return. The "risk discount variant" of the CAPM is also applicable if

- in the case of unlisted companies, there are no historical share price returns to calculate the beta factor, or
- historical returns cannot be regarded as representative for the future, for example, due to capital market imperfections or a strategic decision, like change in the business model.

The risk discount variant or certainty equivalence variant of the CAPM is as follows:

$$(4) \qquad Value(\widetilde{CF}_1) = \frac{E(\widetilde{CF}_1) - \rho(\widetilde{CF}_1, \tilde{r}_M) \cdot \sigma(\widetilde{CF}_1) \cdot \frac{(r_m^e - r_f)}{\sigma(\tilde{r}_m)}}{1+r_f}$$

with ρ as correlation coefficient of the uncertain cash flow and the uncertain return of the market $\widetilde{r}_m$, $\sigma(\widetilde{CF})$ as standard deviation of the expected cash flows (scope of risk expressed in monetary units) and $r_m^e = E(\widetilde{r}_m)$ as expected return of the market portfolio (see *Robichek/Myers*, 1966; *Rubinstein*, 1973 and *Gleißner/Wolfrum*, 2009).

A future-oriented calculation of the correlation ρ is possible either through a so-called "risk factor approach", which models joint influencing factors on $\widetilde{CF}_1$ and the uncertain return of the market $\widetilde{r}_M$ (e.g., economic situation, exchange rate, and oil price) or through a statistical analysis of historical data. It cannot be assumed that historical stock returns, which may also be influenced by psychological factors or momentum trading strategies, show the valuation-relevant risk of the cash flows to be valued (*Dirrigl*, 2009).

In contrast to the traditional CAPM return equation, the variant shown is also applicable to negative cash flows. For communication purposes, the valuation result can also be converted into a cost of capital rate (or an implicit beta factor).

The valuation equation for the risk discount variant of the CAPM can be derived using a robust replication approach even without the restrictive assumptions of the CAPM (see *Gleißner/Wolfrum*, 2009 and *Dorfleitner/Gleißner*, 2018).

4. Deriving the valuation equation and cost of capital from a risk analysis using incomplete replication

4.1 Deriving the valuation equation using incomplete replication

In the following, a so-called incomplete replication approach ("duplication") is used to show how concrete valuation equations (and thus the market price of the risk λ_{CE} can be derived)[27]. Later in 4.2 we will derive the cost of capital.

It is of fundamental importance - and a key advantage - that the following valuation methodology, and the cost of capital derived later in section 4.2, are

[24] This is only correct in context of the market approach for a well-diversified shareholder and for cash flows with a negative correlation to the market returns.

[25] For derivation see *Gleißner/Wolfrum*, 2009.

[26] It is worth to mention that a risk measure $R(\widetilde{CF})$ e.g. $\sigma(\widetilde{CF})$ is not necessary if it is intended to get implied cost of capital (*Bini*, 2018).

[27] *Gleißner*, 2011 and *Dorfleitner/Gleißner*, 2018.

based on only a few, and less restrictive, assumptions that open up a broad field of application:

First assumption:

Two cash flows at the same time have the same value for the valuation subject if they match the expected value and the risk measure chosen by the valuation subject.

Second assumption:

For the subject of the valuation, a[28] risk-free investment with an interest rate r_f and a risk-bearing investment option with an uncertain return $\widetilde{r}_M$ (e.g. a broad empirical market portfolio) are available as alternative investment opportunities.

That's all. In particular, no further assumptions about the capital market are required (this does not have to be arbitrage-free or complete). Further assumptions about the subject of the valuation are required. It does not need to be perfectly rational nor perfectly diversified (due to ancillary assumptions, such as in CAPM).

In particular, no utility function of the evaluation subject must be known because its risk preference only manifests itself in the choice of the risk measure (which, incidentally, is similar in this respect to the CAPM), which specifically underlays a (μ, σ) i.e. a special case of the here generally accepted (μ, R)

In order to determine the value of an uncertain cash flow $\widetilde{CF}_A$ of an investment A in a one-period model, an (incomplete) replication that is in line with expectations and is risk-adequate is carried out. Two investment options should be available for this purpose:

- the (empirical) market portfolio[29] with an uncertain return $\widetilde{r}_M$ and

- a risk-free investment with the interest rate r_f.

It is important to note that in contrast to CAPM or valuation methods based on the assumption of an arbitrage-free capital market[30] this valuation approach does not require any customary, restrictive (and less realistic) assumptions about the capital market. This is a significant advantage of the method explained here. With regard to the capital market, it is only assumed that there is a[31] risk-free investment opportunity and a risky investment opportunity (for example, the ability to invest in a broad market index, such as the MSCI All Country). In particular, it is not necessary to assume that the capital market is perfect, complete or arbitrage-free[32]. No assumptions are required regarding e.g. the absence of taxes or transaction costs. The risky investment opportunity, which can be understood as an "empirical market portfolio", need not have any other condition than that considered by the valuation subject as an investment opportunity[33].

The market portfolio in this context is nothing more than a portfolio of uncertain assets that exist (and can be invested) in the real world.

It is a fundamental advantage of the methodology proposed here that it does not require any restrictive assumptions about (1) the capital market or (2) the behavior of the valuation subject (as explained above, the latter is not necessarily the - in reality non-existent – homo economicus, who, however, chooses operationalized optimal behavior, acts only according to the central assumption 1 above).

Unrealistic and restrictive assumptions are not required for deriving the valuation equations. In particular, the application of the valuation method also allows for constellations in which no sale of the company is envisaged at all (as discussed in the introduction situation of a strategy assessment).

In contrast, e.g. there are no assumptions for the CAPM that would imply that

- Value and price are basically the same,
- The valuation subjects would have perfectly diversified portfolios (and therefore would only bear systematic risks, as in the CAPM).

Value and price can differ so very well in the assumption system made here and valuation subjects - as in reality – are free to have diversified portfolios or not. Due to the lack of the need to use restrictive assumptions, it is possible in particular to cover existing constellations for the valuation which otherwise cannot be assessed (e.g. the evaluation of strategic options for action of an entrepreneur as a valuation subject who owns all his assets in his own company and thus carries company-specific risks).

It is easy to calculate the value (CF) = $x + y$. The amount of capital x invested in the market portfolio and the amount of capital y invested in the risk-free investment is exactly enough that the risk of this portfolio corresponds to the risk of the uncertain cash flow $\widetilde{CF}_A$. The risk is measured by a suitable risk measure $R(\widetilde{CF}_A)$, such as standard deviation, value-at-risk or conditional value-at-risk. The risk measure can generally be selected by the valuation subject and is an expression of the risk perception. In addition to the risk measure of the standard deviation which is usual in capital market-oriented valuation (especially the

[28] At least, so to speak.

[29] This is an "empirical" market portfolio (like a stock market index). Not necessarily the theoretical market portfolio based on the CAPM-Assumptions.

[30] No-arbitrage conditions.

[31] At least, so to speak.

[32] See for an explanation of the terms and their relationship, *Friedrich*, 2015, pp. 13.

[33] It is therefore a "real" investment opportunity and not a model construct, such as the market portfolio at *Markowitz* (1952) or within the framework of the CAPM. The assumption that the (empirical) market portfolio can be invested corresponds to the idea of "availability" in *Richter*, 2005, p. 22.

CAPM), downside risk measures can also be used. With these downside risk measures, risk is expressed as “possible loss” or the utilization of a risk coverage potential (equity and liquidity reserve) that is scarce in reality. The risk measure should be homogeneous and translation- or position-invariant[34].

$$(5)\qquad R(\widetilde{CF}_A) = R\left(x\cdot(1+\tilde{r}_M)+y\cdot(1+r_f)\right)$$

The expected value of the repayment of the investment in the market portfolio and the risk-free investment should correspond to the expected value $E(\widetilde{CF}_A)$.

$$(6)\qquad \begin{aligned} E(\widetilde{CF}_A) &= E\left(x\cdot(1+\tilde{r}_M))+y\cdot(1+r_f)\right)= \\ &= x\cdot(1+E(\tilde{r}_M))+y\cdot(1+r_f) \end{aligned}$$

The value of the risky cash flow $\widetilde{CF}_A$ corresponds to the sum of the two investments x and y. The same risk and the same expected value imply the same value.

$$(7)\qquad Value(\widetilde{CF}_A) = x + y$$

The replication equation can be derived from equations (6) and (7)[35].

$$(8)\qquad R(\widetilde{CF}_A) = R\left(x\cdot(\tilde{r}_M - E(\tilde{r}_M)) + E(\widetilde{CF}_A)\right)$$

If the risk measure is known, this equation can be solved and thus evaluated. It is important to know whether this is a position-dependent risk measure (such as the value-at-risk or the conditional value-at-risk) or a position-independent risk measure (such as the standard deviation or the deviation value-at-risk). The deviation value-at-risk or relative value-at-risk is defined as $DVaR_\alpha(\widetilde{CF}_A) = E(\widetilde{CF}_A) + VaR_\alpha(\widetilde{CF}_A)$ with a as confidence level (e.g. a=99%).

Since cash flows often cannot be described by normal or log-normal distributions (e.g., because of fat tails), downside risk measures are gaining in importance.

In the following, only position-independent risk measures such as standard deviationare considered more closely, since they are seen as a measure of planning reliability or the extent of possible plan deviations (from the expected value)[36]. This applies to these (see *Rockafellar/Uryasev/Zabarankin*, 2002):

$$(9)\qquad R(a+b\cdot\widetilde{CF}_A) = b\cdot R(\widetilde{CF}_A)$$

With equation (9) equation (8) simplifies to

$$(10)\qquad R(\widetilde{CF}_A) = x\cdot R(\tilde{r}_m)$$

For the value one obtains by transformations (and by neglecting a time index)[37]

$$(11)\qquad Value(\widetilde{CF}_A) = \frac{E(\widetilde{CF}_A) - \frac{E(\tilde{r}_m)-r_f}{R(\tilde{r}_m)}R(\widetilde{CF}_A)}{1+r_f} = \frac{E(\widetilde{CF}_A) - \lambda_{CE} * R(\widetilde{CF}_A)}{1+r_f}$$

with $\lambda_{CE} = \dfrac{E(\tilde{r}_m)-r_f}{R(\tilde{r}_m)}$

A special variant of equation (3) has thus been derived[38]. The market price of the risk λ shows how much more return per unit of risk can be expected for the alternative investments under consideration.

In the simplest case, the risk discount corresponds to the product of the risk premium and the risk volume (e.g. “equity requirement” as a risk measure based on value-at-risk).

Now assume the risk measure $R(\widetilde{CF})$ is the standard deviation and so $R(a+b\cdot\widetilde{CF}_A = b\cdot\sigma(\widetilde{CF}_A)$. Now the following equation shows how is the value of the cash flow $\widetilde{CF}_A$.

$$(12)\qquad Value_0(\widetilde{CF}_A) = x + y = \frac{E(\widetilde{CF}_A) - \sigma(\widetilde{CF}_A)\cdot\frac{E(\tilde{r}_m)-r_f}{\sigma(\tilde{r}_m)}}{1+r_f}$$

The cost of capital (c) is thus implicitly the ratio of the cost of $E(\widetilde{CF}_A)$ *to* $Value_0(\widetilde{CF}_A)$ which will be discussed later in section 4.2.

Until now, it has been assumed that the cash flow from investment A and the market portfolio is fully correlated, i.e., that the correlation coefficient $\rho_{AM} = 1$ or investment A is the only asset.

As a rule, however, this assumption will not be fulfilled and thus diversification possibilities will be available so that only the non-diversifiable portion of the risk (the systematic risk) of the cash flow is relevant for the valuation.

This reduces the valuation-relevant risk of the cash flow by multiplying the standard deviation by $\rho_{AM} = \rho(\widetilde{CF}, \widetilde{r}_M)$, so that the following equation results:

$$(13)\qquad Value_0(\widetilde{CF}_A) = \frac{E(\widetilde{CF}_A) - \rho_{AM}\cdot\sigma(\widetilde{CF}_A)\cdot\frac{E(\tilde{r}_m)-r_f}{\sigma(\tilde{r}_m)}}{1+r_f} = \frac{E(\widetilde{CF}_A) - \lambda\cdot\sigma(\widetilde{CF}_A)\cdot\rho_{AM}}{1+r_f}$$

Equation (14) corresponds to the certain equivalence equation of the CAPM (3)[39]. The following conditions apply:

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34 Position-independent. See *Dorfleitner/Gleißner*, 2018.

35 See *Dorfleitner/Gleißner*, 2018.

36 See *Dorfleitner/Gleißner*, 2018 for translation- invariant risk measures.

37 $\widetilde{CF} = \widetilde{CF}_1$ is considered to be the cash flow of period 1. Period 1 is between time t=0 and t=1. Valuation date is t=0.

38 See *Gleißner/Wolfrum*, 2009.

39 See *Robichek/Myers*, 1966; *Rubinstein*, 1973.

- the risk measure is the standard deviation,
- only non-diversifiable, systematic risks are assessed, and
- there are homogeneous expectations, i.e., the cash flow is valued by the capital market according to the planning ($\sigma(\widetilde{CF}_A) = \sigma_A$).

It should be noted that the replication equations do not conflict with CAPM if the same assumptions are made as in CAPM and in this case the risk measure ("capital requirement", CVaR or VaR) contains exactly the same information as the standard deviation and the beta factor (see *Mai*, 2006, on the relationship with the traditional CAPM return equation, specifically on the assumption of proportionality of cash flow and value fluctuations).

The replication methodology can also be extended to multi-period cash flows[40].

4.2 Deriving the cost of capital from the valuation equation using incomplete replication

The procedures described in section 4.1 allow the risk-adjusted measurement of uncertain cash flows (in one or more periods). However, valuation using a risk discount in the numerator, i.e., the calculation of certainty equivalents, is unusual in valuation practice. The previously explained (semi-investment theoretical) valuation based on "incomplete replication" can, however, also be directly linked to the discounted cash flow (DCF) methods known in practice. For this purpose, it is necessary to determine the cost of capital (discount rate) of the DCF methods using the methods explained in section 4.1.

The bridge from the aggregated total risk, e.g. expressed by the standard deviation of the cash flow $\sigma(\widetilde{CF})$, to the company value, is precisely the cost of capital (or certainty equivalents). In contrast to the traditional "capital market-oriented" valuation, the cost of capital in a risk simulation can be derived directly from the earnings risk and not from historical stock return fluctuations (as is usually the case with the beta factor of the CAPM; see *Gleißner*, 2011 and 2014). The results of a risk analysis are used, on the one hand, to obtain expected cash flow values and, on the other hand, to derive the cost of capital rates consistently (the consistency between the expected value of the cash flows in the numerator and the cost of capital rates in the denominator is a notable advantage of the methodology explained). Such a discount rate, which is often assumed to be constant, can be derived as a risk measure from the standard deviation of the cash flow, for example. It obviously applies:

$$Value_0(\widetilde{CF}_A) = \frac{E(\widetilde{CF}_A)}{1+c} \tag{14}$$

If one resolves this equation with equation (14) for the value V after c, one obtains the risk-adequate cost of capital. If $\widetilde{CF}$ is the operating free cash flow (oFCF), c is the weighted cost of capital (WACC)); if $\widetilde{CF}$ is the flow to equity (FtE), c is the cost of equity.

Based on the risk-free interest rate, the following equation for the risk-adequate capitalization rate (cost of capital) is obtained[41]:

$$c = \frac{1+r_f}{1-\lambda\cdot\frac{\sigma(\widetilde{CF}_A)}{E(\widetilde{CF}_A)}\cdot d} - 1 = \frac{1+r_f}{1-\lambda\cdot V\cdot d} - 1 \tag{15}$$

The ratio of cash flow risk $\sigma(\widetilde{CF}_A)$ to expected cash flow $E(\widetilde{CF}_A)$ is the coefficient of variation V. The variable λ shows the excess return per unit of risk (*Sharpe* Ratio).

$$\lambda = \frac{Market\ Risk\ Premium}{\sigma(\tilde{r}_m)} = \frac{E(\tilde{r}_m)-r_f}{\sigma(\tilde{r}_m)} \tag{16}$$

λ is dependent on the expected return of the market index, its standard deviation and the risk-free rate of return and expresses the risk/return profile of the alternative investments: to value means to compare (*Moxter*, 1983). As the owners do not necessarily bear all the risks of the company, the risk diversification factor d must also be taken into account. It shows the proportion of risks of a company that the owner has to bear in equation (16)[42].

An estimate of the degree of risk diversification d can be derived by the correlation of the (trend-adjusted) earnings (or earnings growth) of the company to the earnings of all companies in the market index. The risk diversification factor d implicitly follows from the simulation-based risk aggregation if exogenous risk factors are considered independently to record the systematic, cross-company risk[43]. Under the special assumptions of the CAPM, d conforms to a correlation with the return on the market portfolio.

Equation (16) can be used for different definitions of cash flows $\widetilde{CF}_A$. If flow to equity is used as cash flow, the cost of equity is obtained. If the operating free cash flow is used, the weighted average cost of capital (WACC) is obtained. The WACC results

[40] See *Dorfleitner/Gleißner*, 2018.

[41] For $\lambda\cdot\frac{\sigma(\widetilde{CF}_A)}{E(\widetilde{CF}_A)}\cdot d < 1$

[42] It is the proportion of $R(\widetilde{CF})$ to additional ("incremental") risk in the portfolio of the owner caused by the company (see *Gleißner*, 2011 and *Tasche/Tibiletti*, 2003).

[43] "Risk factor model"; see *Gleißner*, 2017a, pp. 261-263.

"directly", without first having to calculate cost of equity and cost of debt and weight them appropriately. The total extent of the risks determines the total cost of capital (only in a second step are the total risks divided between equity and debt capital providers, which determine the cost of equity and cost of debt). Determining the total cost of capital (WACC) in this way is comparatively simple and there is no need for leveraging or deleveraging when calculating the cost of equity.

It should be noted that it is not necessary to calculate the cost of capital only for a representative period and to assume it to be constant for all periods for the sake of simplicity. Of course, it is also possible to calculate periodic cost of capital. In addition to periodic cost of capital, it is also possible, and useful in many valuation cases, to calculate two different cost of capital: a cost of capital c_1 for the detailed planning period and a cost of capital c_2 for the continuation period. This is particularly appropriate if, in the detailed planning period, the risk-return profile of the company, and thus the coefficient of variation V, still differ significantly from that in the continuation period. This is particularly the case if, for example, a young company has significantly higher risks at the beginning of its existence than later when it is established (i.e. in the continuation period).

4.3 Risk analysis and risk aggregation using Monte Carlo simulation

The identification and quantification of the company's risks (opportunities and threats) must be the basis for the risk-appropriate evaluation of a company.

As a result, the risk analysis and risk aggregation - as shown above - leads to cost of capital that express the risk-adjusted requirement for the return on a project, business unit or company (e.g., for the calculation of a discounted cash flow DCF or Economic Value Added EVA). In addition to the risk measure of the standard deviation, which is based on a normal distribution and is used in CAPM, there are other risk measures. These risk measures are often better suited to describe the actual risk in the company. In order to determine suitable risk measures for company valuation, the actual risks in the company must be determined. This is done with the help of a risk analysis. Then it has to be examined how the risks are related to each other and how they affect the cash flows and thus the company value. This is done on the basis of a Monte Carlo simulation. The results of the Monte Carlo simulation can be used to calculate suitable risk measures. These risk measures are then incorporated into the company valuation using the certainty equivalence method and are expressed by the variable $R(\widetilde{CF})$

4.3.1 Risk analysis of corporate risks

The first step in risk analysis is the identification of risks, which can be structured as follows:

(1) Strategy and strategic risks

Strategic risks are the risks arising from the threat to the company's most significant potential for success.

(2) Controlling, operational planning and budgeting risks

In controlling, business planning or budgeting, certain assumptions are made (for example, with regard to the growth rate of the economy, exchange rates and successes in sales activities). All uncertain planning assumptions show a risk because plan deviations can occur. The causes of plan deviations show the effects of existing risks.

(3) Risk workshops (risk assessment) on performance risks

Certain types of risk are best identified in a workshop through critical discussions. These include, in particular, operational risks, legal risks, political risks, and risks arising from support services (e.g., IT).

For the quantitative description of a risk, a probability distribution can be used that describes the effects of a risk on earnings in a period (e.g., year). A more differentiated consideration is possible if a risk is described by (1) a probability distribution for the frequency of the occurrence of the risk in a period and (2) a probability distribution for the amount of damage per occurred risk event.

4.3.2 Risk aggregation using Monte Carlo simulation

It is not individual risks but the aggregated overall risk scope that is decisive for assessing a company's (free) risk-bearing capacity and the degree of threat to its continued existence. Aggregation across all individual risks and over time is therefore necessary. Since only quantified risks can be aggregated, all relevant risks must be quantified. By aggregating the quantified risks in the context of planning, it is examined what effects these have on future earnings, future cash flows, the key financial indicators, credit agreements (covenants), the rating, and thus on the enterprise value. For example, it is necessary to calculate the probability that risks (e.g., an economic downturn in connection with a failed investment project) could cause the company's future rating to fall below a level (B rating) necessary for the company's ability to service its debt.

The aggregation of risks in the context of corporate planning requires the use of simulation methods (Monte Carlo simulation) because risks - unlike costs - cannot be added together, at least if special cases (normal distributions) are excluded. Furthermore, risks in an integrated planning model must also be aggregated over several years to identify serious crises over time. Simulation methods are the further development of the well-known scenario analysis techniques[44]. Monte Carlo simulation is used to analyze a large representative number of risk-related possible future scenarios (planning scenarios) in risk aggregation. In this way, a frequency distribution and thus a realistic range of future cash flows and returns are shown, i.e., the planning reliability or extent of possible negative deviations from the plan.

4.3.3 Risk measures

In addition to the quantitative description of risks, the calculation of risk measures (R) is another sub-task in risk quantification[45]. The term risk measure is a collective term for statistical measures that make it possible to describe the uncertainty of an event quantitatively. A risk measure maps a frequency or probability distribution to a real number. A risk measure expresses the scope of risk of a distribution in a number that can then be used for further economic and application-oriented calculations. Risk measures are necessary to enable simple "calculating with risks" (as shown in section 4). They thus serve to transform risk or uncertainty.

A distinction is made between position-dependent (position-invariant) and position-independent risk measures. Position-dependent risk measures, such as the value at risk, are dependent on the expected value. If a position-dependent risk measure is not applied to a random variable $\widetilde{X}$, but to a centered random variable $\widetilde{X} - E(\widetilde{X})$, the result is a position-independent risk measure[46]. Position-independent risk measures (such as the standard deviation or deviation value at risk (DVaR)) describe the extent of plan deviations and are therefore also referred to as deviation measures.

Furthermore, a distinction is made between one-sided and two-sided risk measures. Two-sided risk measures measure deviations from the planned or expected value in both directions, i.e., opportunities and risks. The one-sided risk measures consider only possible deviations in one direction, mostly possible negative plan deviations.

For the derivation of the evaluation equations, it is assumed, as explained above, that the risk measure (a) is homogeneous and (b) is either translational or position-invariant, and therefore the following applies accordingly:

- positive homogeneity (PH) is defined by $R(a\widetilde{X}) = aR(\widetilde{X})$,
- translation invariance (TI) is defined by $R(\widetilde{X} + a) = R(\widetilde{X}) - a$,
- position invariance (PI) is defined by $R(\widetilde{X} + a) = R(\widetilde{X})$

with $a \in \mathbb{R}$ and a >0

5. Insolvency risk and rating

Previously, this section explained how the risks (opportunities and threats) affect the expected value of cash flows and the cost of capital. In real, incomplete capital markets with rating and financing restrictions, there is a further impact of risks that is discussed below. A particularly unfavorable combination of individual risks can arise scenarios that lead to the insolvency of the company and thus to the interruption of the cash flow of the (previous) owners. This risk of insolvency has so far received little attention in valuation practice, although it can have considerable effects on the value of the company.

It should be noted that the insolvency risk, especially the probability of insolvency p, influences the expected value of the cash flows and their development over time[47].

In the detailed planning phase, the probability of insolvency must be taken into account directly when determining the expected values (as a scenario with, as a rule, no return to the owners). In general, it is advisable to map insolvency scenarios in detail in a stochastic event space or in the paths of a simulation model even in the continuation phase.

In addition to considering the insolvency scenario in the detailed planning, it should be noted that insolvency can occur in any year of the continuation phase. An approach that is partly implemented in valuation practice is the evaluation of an insolvency scenario for the expected result. Even if this may already sensitize to the possibility of insolvency, considerable problems remain: On the one hand, the estimated probability of insolvency is usually not rating and planning consistent, on the other hand, it is often ignored that insolvency is possible every year, so that there are many insolvency scenarios - and in the long term, insolvency is a scenario with a high probability.

If it is assumed for the continuation phase when determining the terminal value that the probability of insolvency - corresponding to the steady state in the terminal value formula - remains constant, it leads (under otherwise identical conditions) over time to continuously declining expected cash flows.

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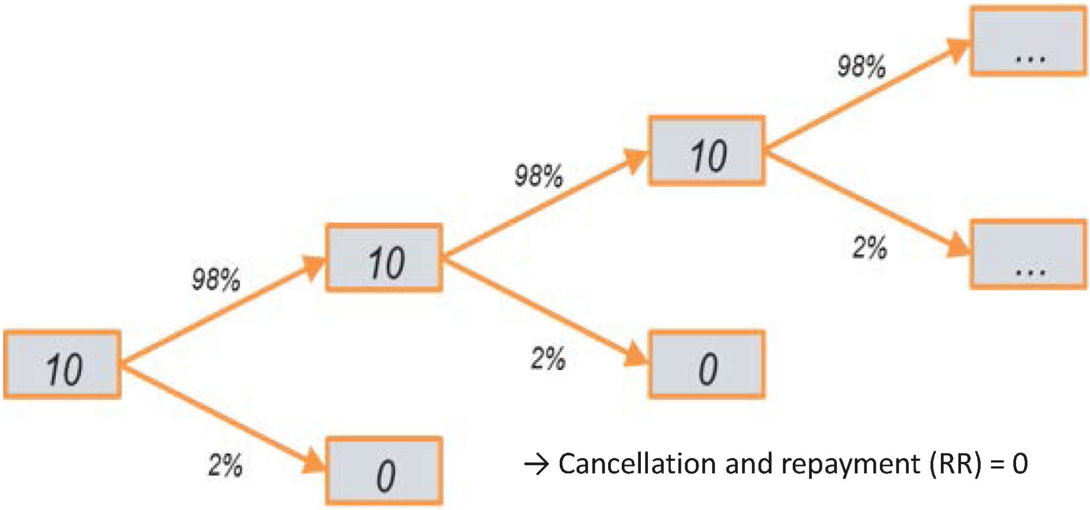
[44] See *Grisar/Meyer*, 2015 and 2016 on significance.

[45] See cf. *Gleißner*, 2017a and *Artzner et al.*, 1999, *Pedersen/Satchell*, 1998, *Albrecht/Maurer*, 2005 and *Brandtner*, 2012.

[46] See *Pedersen/Satchell*, 1998.

[47] See *Gleißner*, 2010 and *Friedrich*, 2015 and 2016 and *Lahmann/Schreiter/Schwetzler*, 2018.

Graph 2: Implications of a probability of defaults (p=2%)



In the long term - in the continuation phase – p has the effect of a negative growth rate[48] (see graph 2), which must be taken into account when calculating the terminal value (TV)[49]. This applies here:

(17) $$Value^{TV} = \frac{E(\widetilde{CF})\cdot(1-p)}{c+p}$$

This also applies if cost of capital or discount rates (c) are calculated according to the CAPM.

With a growth rate[50] (g), the (conditional) expected values of the cash flows $E(\widetilde{CF})$[51] and a discount rate (c), the following equation results for the company value (Value) in the continuation phase (terminal value) as a function of the insolvency probability (p)[52]:

(18) $$Value = \sum_{t=0}^{\infty} \frac{E(\widetilde{CF})\cdot(1-p)\cdot(1+g)^t}{(1+c)^t} = \frac{E(\widetilde{CF})\cdot(1-p)\cdot(1+g)}{c-g+p\cdot(1+g)}$$

The value of a company (or its terminal value) with g = 0 is then:

(19) $$Value = \frac{E(\widetilde{CF})\cdot(1-p)}{c+p}$$

When determining an infinite series (Gordon Shapiro model), the insolvency probability (just like the growth rate) actually appears in the numerator in each individual period (see equation (17)). However, the dissolution of the series leads to the fact that the probability of insolvency (as well as the growth rate) mathematically "migrates" into the denominator. This does not mean, however, that double counting would occur or that the probability of insolvency would become a component of the discount rate. In the continuation phase, the probability of insolvency thus largely acts like a "negative growth rate" - but is not part of the cost of capital.

Anyone who accepts the recording of a growth rate in the terminal value must also accept the consideration of the probability of insolvency derived from the same assumption system. The above-mentioned "pragmatic" recording of the possibility of insolvency within the framework of the usual (deterministic) "terminal value formula" is not without alternatives. A more precise recording of the risks and stochastic dependencies, also between the individual periods, can be achieved e.g. by binomial models (Friedrich, 2015)[53] and especially by flexible stochastic planning models and Monte Carlo simulation. When calculating the expected values in the simulation, the insolvency scenarios are recorded and a closed "terminal value formula" is practically unnecessary if one simulates many years of the future. Nevertheless, as explained above, pragmatic solutions certainly also have practical advantages.

[48] See *Shaffer*, 2006, *Gleißner*, 2010; *Knabe*, 2012; *Saha/Malkiel*, 2012; *Ihlau/Duscha*, 2019.

[49] See *Gleißner*, 2017c, *Knabe*, 2012 and *Saha/Malkiel*, 2012 and *Lahmann/Schreiter/Schwetzler*, 2018.

[50] On the relationship between w and c in inflation-, accumulation- and tax-indexed (endogenous) growth see *Tschöpel/Wiese/Willershausen*, 2010.

[51] Without insolvency (conditional expected value) and period-invariant probability of insolvency (here for T, i.e. after detailed planning phase).

[52] $E(\widetilde{CF})$ is the expected value of growth and probability of insolvency. If $E(\widetilde{CF})$ is interpreted as cash flow before probability of insolvency, $(1 + g)$ is omitted.

[53] In addition, one can immediately see with binomial models by *Friedrich*, 2015, that, as is usual with such (simple) binomial models, no negative free cash flows can occur, which is unrealistic. Insolvencies naturally occur especially with negative free cash flows. The impossibility of depicting negative cash flows in the simple binomial model results from the fact that in the binomial tree the last cash flow is multiplied by 1.4 (up scenario) with a previously given probability (e.g. p = 60%) or by 0.8 (down scenario) with a probability of $(1 - p)$.

6. Case study: From CAPM to risk-adequate assessment

6.1 Introduction

The explanations above will be illustrated in the following with a small example. The transition from "traditional" planning, which here is based on the assumption of (ambitious) planned values of the company and discount rates calculated using CAPM, takes place in three steps.

1. The systematic analysis of existing risks allows a transparent reconciliation of the usual planned values with the expected values relevant to valuation, which will be realised "on average". This creates transparency with regard to the essential, even uncertain planning assumptions and an adequate consideration of a risk overhang.
2. The probability of insolvency expressed by the rating can be assessed by means of a key financial figures rating and the evaluation of combined effects of risks (Monte Carlo simulation). The often ignored value driver "probability of insolvency (insolvency risk)" is taken into account in the implications for the company value and thus takes into account the fact that, contrary to the usual assumption, companies do not exist forever (see section 5).
3. The transparency created by risk analysis and risk simulation (risk aggregation) with regard to planning security and thus the aggregated cash flow risk (cash flow volatility) makes it possible to derive risk-adjusted cost of capital. Expected values of cash flows ("numerator") and discount interest rate ("denominator") are thus determined consistently and the problems of the low informative capacity of CAPM cost of capital (due to capital market imperfections) are avoided. This enables a risk-adjusted valuation, i.e. a calculation taking into account the risks of a company's future earnings and cash flows.

6.2 Initial Situation: CAPM and planning values (corporate planning)

The valuation of the company is based on a two-year detailed planning period (t = 1,2) whereby the second period is regarded as representative for the future[54]. The enterprise has planned the cash flow to equity to be discounted. The long-term growth rate is assumed to be g = 0 and insolvency risks are neglected. The following assumptions are made about the parameters of the environment:

- r_f= 3% (for all periods)
- $r_m^e = 5\%$ (market risk premium)
- ß= 0.75 (calculated with a market price of the risk $\lambda = 0.25$)[55]

From the information provided, the following time-invariant cost of capital results.

(23) $c = r_f + \beta \cdot (r_m^e - r_f) = 3\% + 0.75 \cdot 5\% = 6.75\%$

The following applies to the value

(24) $\text{Value} = \sum_{t=1}^{\infty} \frac{E(\widetilde{CF}_t)}{(1+c)^t}.$

Table 1: Company valuation based on planned values and CAPM

T	1	2	TV	NPV of the cash flows and TV
Cash flow (planned)	10	15	(15 ...)	
c (CAPM, Beta)	6.75%	6.75%	6.75%	
Value	9.37	13.16	195.01[56]	**217.54**

On the basis of the cash flows and terminal value shown in table 1, the company value is calculated as

(25) $Value_1 = 217.54$ Euro

6.3 First step: Transfer from plan values to expected values

The discounted cash flow methods are based on expected cash flows. In order to calculate these, the results of the analysis of chances and risks of the company are used. In particular, uncertain planning assumptions, which form the basis for the cash flow forecast in table 1, are considered and described using appropriate probability distributions. Without further explanation of details, it is assumed that risk analysis and risk aggregation (Monte Carlo simulation) result in a threats overhang and thus lower expected values compared with the planned values.

All other information is unchanged, i.e. the cost of capital rate c = 6.75% derived from CAPM is still used. The Monte Carlo simulation also produces a quantification of the cash flow risk, in the example here a coefficient of variation of V = 0.35, which, however, is not (yet) included in the valuation (see step 3 in 6.5).

[54] For t = 3, 4, ..., 8.

[55] It shall apply to company i: $c = r_f + \lambda \cdot \sigma_i \cdot \rho$ with σ_i as the standard deviation of the stock return of i.

[56] The present value in t = 0 of the TV in t = 2 is 195.01 = 15 / ((0.0675)(1 + 0,0675)^2).

Table 2: Company valuation based on expected values and CAPM

T	1	2	TV	NPV of the cash flows and TV
Cash flow (planned)	10	15	15	
Cash flow (expected)	9	13	13	
c (CAPM, Beta)	6.75%	6.75%	6.75%	
Value	8.43	11.41	169.01	**188.85**

Taking into account the effects of the opportunities and threats on the expected value of the cash flows, the resulting value is now $Value_2$ = 188.85

6.4 Second step: Consideration of the effects of insolvency risk

In step 1, the company's earnings risks were taken into account. However, no account was taken of the fact that risk-related future scenarios could arise for the company, which could lead to insolvency and thus to the discontinuation of the cash flows for the owners (as the valuation subject). Now it is taken into account that insolvency risks influence both the expected value of the cash flows in each period of the detailed planning phase and the expected value in the continuation phase (t > 2). Further effects of the insolvency risk, e.g. on the tax shield, are neglected. It is also assumed that the implications of the probability of insolvency *p* expressed by the rating are already included in the interest rates and thus in the cost of debt (and thus in the expected values of the cash flows). In general, it is also necessary to adjust interest rates and cost of debt to the rating.

In the case study, the probability of insolvency *p* is estimated based on financial ratios, i.e. equity ratio 25% and return on capital employed 10%. (The Monte Carlo simulation carried out for risk aggregation serves to check the plausibility of the probability of insolvency). Furthermore, an insolvency probability of *p* = 1.55% is assumed[57].

This results in the following company valuation:

Table 3: Company valuation based on expected values, CAPM, and insolvency risk

T	1	2	TV	NPV of the cash flows and TV
Cash flow (planned)	10	15	15	
Cash flow (expected)	9	13	13	
probability of survival	98.45%	96.92%	95.42%	sinking with p
Cash flow (expected, incl. insolvency risk)	8.86	12.60	12.40	
c (CAPM, Beta)	6.75%	6.75%	6.75%	
Value	8.43	11.41	131.15	**150.51**

The company value is reduced to $Value_3$ = 150.51 Euro due to the consideration of insolvency risk.

6.5 Third step: Calculation of cost of capital based on earnings risk (coefficient of variation of earnings)

As already mentioned, the coefficient of variation of the returns is - according to the simulation - *V* = 35%. The risk diversification factor here is *d* = 0.5, which corresponds precisely to the correlation between the return on the shares of the valuation object and the return on the market portfolio.

With the results from risk analysis and risk simulation in step 1, the coefficient of variation of the returns was calculated in addition to the adjustment of the planned values, but has not yet been taken into account. The coefficient of variation is a measure of the overall scope of risk (extent of possible deviations from the plan) With equation (15) explained above, information about the risks of the company - instead of information about the risks of the company's shares - is now used as the basis for deriving the discount rate. The following applies accordingly

$$(26) \qquad c = \frac{1+r_f}{1-\lambda \cdot V \cdot d} - 1 = \frac{1+0.03}{1-0.25 \cdot 0.35 \cdot 0.5} - 1 = 7.71\%$$

In this third step, the company value is now determined with a cost of capital *c* corresponding to the earnings risks.

[57] It is based on an empirically determined simple formula for estimating the probability of insolvency $P = \frac{0.265}{1+e^{-0.41+7.42 \cdot equity\ ratio+11.2 \cdot ROCE}}$ (see *Gleißner*, 2017a, pp. 336-338 with reference to the basic research projects).

Table 4: The final valuation

T	1	2	TV	NPV of the cash flows and TV
Cash flow (planned)	10	15	15	
Cash flow (expected)	9	13	13	
probability of survival	98.45%	96.92%	95.42%	sinking with p
Cash flow (expected, incl. insolvency risk)	8.86	12.60	12.40	
c (earnings risks *V*)	7.71%	7.71%	7.71%	
Value	8.23	10.86	115.47	**134.56**

The resulting value is now $Value_4$ = 134.56 Euros.

Now the information on the risk profile of the company as a whole is adequately taken into account in the company valuation. It should be mentioned that the adjustment according to steps 1 and 2 is also necessary if the valuation of perfect capital markets and in particular the validity of the assumptions of the CAPM are assumed.

In the example, in comparison to the initial situation, the enterprise value decreases with every further step. This is not necessarily the case. Thus, there are constellations in which existing opportunities outweigh existing dangers and thus the expected value is higher than a (conservative) plan value. The consideration of the probability of insolvency (p), contrary to the first impression, does not necessarily lead to a lower enterprise value. This is because, in valuation practice, the growth rate g applied in view of economic growth for a company's long-term profit growth (in the continuation phase) is implicitly offset by a 'typed' probability of default ($\bar{p}'$).

Empirical studies[58] show typical growth rates in the order of 0 to 0.5% in the continuation phase. This is much less than the inflation rate alone (excluding real economic growth) and can only be explained by assuming it as an "insolvency-risk-adjusted" growth rate with a typical probability of insolvency (of, for example, 1%) already deducted. The implication for the valuation of different companies is clear: if implicit (and non-transparent) is valued with a medium probability of default, which is offset against the growth rate, it leads to advantages and disadvantages for certain companies: companies with a below-average probability of default have a higher value compared with the traditional approach. The approach tends to be too low, while those with an increased probability of default are too high. *Campbell*, *Hilscher* and *Szilagyi* (2008) show, for example, that companies with a very good rating on the stock exchange generate above-average returns that can be explained if one assumes that the probability of default is ignored, especially in the valuation calculus of most capital market participants, and thus "quality companies" with a very good rating tend to be undervalued and accordingly generate above-average risk-adjusted returns).

7. Summary and outlook

In practice, there are many problems with the valuation of companies, for example due to the often unjustified assumption of perfect capital markets. With risk analysis, Monte Carlo simulation and the method of incomplete replication, instruments exist that take account of the imperfections of the capital market and can also be applied to companies that are not listed on the stock exchange. The valuation-relevant risks are derived by means of risk analysis and risk aggregation, and planning consistency - e.g. via standard deviation or VaR as risk measure - is recorded in the valuation, whereby financing restrictions of the creditors can be taken into account. The detour of obtaining risk information from historical stock returns - instead of from the company itself - is avoided.

Even if CAPM-based valuation is to be applied, the "risk discount variant of CAPM" and the information provided by the risk analysis can be used to ensure that the appraiser is not dependent on historical stock returns that are often missing or not representative for the future. In this respect, the valuation approach also contributes to a new (more accurate) interpretation of the paradigm of value orientation (value based management): orientation towards the interests of the owners, but use of the best available information - and these are not always those of the capital market.

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Residual Income Model and abnormal returns: a comparison to factor styles and sell-side analysts

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We investigate the return forecasting ability of a residual income model based on analysts' estimates and time-varying risk-free rates, equity risk premiums and terminal value growths, in US and Europe, along the period 1995-2018. While the academic literature acknowledged the reliability of the model, practitioners and especially market operators paid scarce attention to it. Therefore, in a framework where market inefficiencies are admitted, a valuation model that shows superior predicting power for returns, at least compared to main market multiples and analysts' recommendations, should be considered in providing better empirical estimates of intrinsic value. We display three major results: a) RIM-based V/P portfolios yield statistically significant alphas relative to market indexes; b) outperform portfolios built through other factors, reporting higher Sharpe ratios and information ratios; c) remarkably beat analysts' buy-sell recommendations. Furthermore, two facts stand out: RIM proves to be extremely effective in signaling overvalued stocks and producing substantial long-short returns; the simpler RIM model studied generates better outcomes than the more complex one.

1. Introduction

The performance of the residual income model as a valuation tool and as a return estimator has been extensively studied in the financial and accounting literature, in particular in the period 1995-2006. Nevertheless, it emerged from recent surveys that the model is still not favored by practitioners in performing valuations, especially among sell-side analysts. This work traces the history of the relevant literature first and then analyzes the return forecasting potential of RIM using a monthly equally-weighted asset allocation, both long-only and long-short (as in standard factor testing) for the period 1995-2018. The models we used to implement a full valuation of stocks in US and Europe and generate portfolio rankings according to a RIM-based V/P, are based on analysts' forward estimates on earnings per share (EPS) and dividends per share (DPS). The structures studied have been essentially two: one with a truncation at the third year of analysts' estimates, one with additional growth of 5 years through sustainable growth. We estimated the cost of capital through CAPM while considering both time-varying risk-free rates, equity risk premiums and terminal growths. Even if our purpose is not to speculate about the efficiency of the market, we simply do not rule out the possibility of obtaining abnormal returns, expecting a long-term convergence between price and value as showed by Lee et al. (1999)[1] like in a co-integrated system. Therefore, if it is possible to exploit abnormal returns with a residual income model, we infer that the model is a good tool for intrinsic value evaluation. First of all, our RIM-based V/P multiple allocation outperformed main market indexes both in US and Europe. In the period 1995-2018, considering yearly compounded returns, long-only top ranked portfolios outperformed the S&P 500 by 4% and 6.5% in US, while outperformed the STOXX 600 by 4.5% and 7.4% in Europe (depending on the type of V/P considered). The monthly alphas produced by RIM-based allocation, against our benchmarks, came in between 0.34% and 0.57% in US and between 0.47% and 0.63% in Europe, with a statistical significance above 95% (details of t-stat for all portfolios at APPENDIX A, Table - A2 and APPENDIX B, Table - B2). Besides, long-short portfolios produced a yearly compounded self-financing return between 7.6% and 12.6% in United States and between 4.1%

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[1] Lee C. M. C., Myers J., Swaminathan B. (1999). What is the intrinsic value of the Dow? *Journal of Finance, 54(5), pp. 1693-1741.*

and 4.6% in Europe. As a second main point we compared the results of RIM-based V/P to other main factors (both trailing and forward) traditionally used as signals of "Value Premium": P/E, P/BV, PEG, ROE, EV/EBITDA, Size. It can be shown that V/P produced consistent higher risk-adjusted returns (Sharpe ratios and information ratios) compared to the majority of other multiples, with the exception of $ROE^{(forward)}$ and EV/EBITDA multiple in Europe, which produced risk adjusted returns (in long-only portfolios) in line with the RIM-based valuation. It is worth to underline that the RIM-based V/P produced the highest differential returns compared to other factors in long-short portfolios. It can be inferred that the model has been extremely good in signaling overvalued stocks. Finally, we studied the performance of analysts' recommendations (through the TP/P multiple) since we used their estimates as inputs of the model. In line with the academic literature on analysts' biases, we identified the multiple TP/P as the worst return predictor and we did not find any significant correlation with other valuation multiples both for US and Europe. While TP/P multiple produced the worst strategy in our sample for the full period (1995-2018) it is important to notice that in the last sub-period (2010-2018) analysts signaled differential target price forecasting ability, especially in Europe. Even though analysts did not produce value through their recommendations in general, they produced financial estimates that enabled us to obtain abnormal returns with RIM. Again, this finding is consistent with one side of academic literature which concludes that analysts' estimates are a better proxy for expected earnings than those from time-series models. Finally, it is worth pointing out that the use of a residual income model-based valuation could remarkably improve the analysts' price target quality. The article is structured in 5 chapters. Chapter 2 illustrates the theory behind the residual income model and its different consideration among academics and practitioners. Chapter 3 traces the three main areas of financial literature that get through this paper: the market efficiency, the factor theory and the relevance of sell-side analysts forecasts. Chapter 4 outlines the method used to collect data and to construct equally-weighted monthly portfolios. Finally, Chapter 5 summarizes the results obtained through our RIM-based V/P multiple across US and Europe. In particular the performances of monthly constructed portfolios (both long-only and long-short) against main market indexes, other factors styles and sell-side analysts' recommendations.

2. Equity valuation with residual income model

The valuation model we employed to estimate the equity value of firms (per share) is the residual income model (RIM), sometimes specified also as "Ohlson model" (OM). While RIM has been extensively studied in the financial and accounting academic literature e.g. on the value-relevance (Barth, Beaver and Landsman, 2001), on the relation between accounting and cash flow based valuations (Penman and Sougiannis, 1998; Courteau, Kao and Richardson, 1999; Penman, 2001), on discrepancy between values and prices (Lee, Myers and Swaminathan, 1999; Ali, Hwang and Trombley, 2003), on the cost of capital and ERP estimates (Gebhardt, Lee and Swaminathan, 2001) and on the relation between risk and return (Penman and Reggiani, 2013) it did not receive the same attention among financial practitioners, at least in the equity research sector, where has been possible to verify its application in equity reports[2]. Looking at a previous study on the matter, by Hand, Coyne, Green and Zhang (2017), it emerges that, among US sell-side equity research analysts, RIM was employed just 1/20 as often as DCF (1/17 in non-US countries) and 1/8 as often as multiples (1/6 in non-US countries). The same research highlights that, among brokerage houses, only Morgan Stanley was a frequent user in equity research reports, which confirms its historical acknowledgement of residual income model as a valuable tool[3-4]. Another research, conducted by Richardson S., Tuna I. and Wysocki P. (2010), highlights that RIM is less frequently used by practitioners compared to academics. According to their survey only 16% of practitioners use RIM frequently, whereas 71% of academics use it frequently. On the other side, 74% of practitioners use earnings multiples frequently, compared to 54% of academics[5].

The main advantages of RIM can be summarized in the following points: it usually implies a lower weight of terminal value on net present value, compared to other valuation models; it can be applied to companies that do not pay dividends or that do not have positive free cash flow in the short term; it safeguards from the risk of overvaluation determined by profits produced

[2] Hand J. R. M., Coyne J., Green J., Zhang X. F. (2017). The use of Residual Income valuation methods by U.S. sell-side equity analysts, *Journal of Financial Reporting*, Spring, 2(1), pp. 1-29.

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[4] Giuliani S. (2005). *Valore d'impresa: rischio e allocazione del capitale, Roma: Aracne, pag. 177.*

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by bigger investments; it is neutral to some earnings management, like cost capitalization; it captures the sources of value not represented in the balance sheet, like intangibles. On the other side the underpinned drawbacks are: it is based on accounting numbers that could be manipulated by management; it assumes that the cost of debt is properly reflected by passive interests, since it uses net profit as an input; it is based on the "clean surplus accounting" relationship [$BVPS_{(t+1)} = BVPS_{(t)} + EPS_{(t+1)} - DPS_{(t+1)}$], which is violated in the case of shares transactions, currency translation, pension adjustments and certain changes in fair value (all changes that refer to "other comprehensive income")[6]. Even though every valuation model with infinite forecasting horizon (and fully consistent assumptions) should provide the same results, we share the view of Penman (1998), who shows how accrual earnings techniques dominate cashflow models in managing the "truncation problem" in valuations with finite horizons[7]. De facto, in our valuations we kept into account analysts estimates up to 3 years of forecasts considering that they are more frequent and usually more followed by market participants, even if sometimes it is possible to find estimates till 5 years.

We tested RIM, as a full valuation model, against factor models (which are usually implemented through multiples), also as we consider it to be one of the most conservative valuation technologies and one of the easiest to implement, using analysts per share estimates as an input. While analysts' estimates frequency on operating accounting data (e.g. EBIT, EBITDA, EBITA, EBITAR) can vary among sectors and companies, since one metric could be more relevant in one industry compared to another, estimates about EPS are available for every company covered by a broker research. Conversely estimates on cash flow items are less accurate than earnings forecasts[8] and are seldom recorded in a consistent way by data providers, especially collecting backwards observations. Finally, we attributed importance to EPS estimates being aware of the research of Fried and Givoly (1982)[9] and Brown et al. (1987)[10], who supported the academic conclusion that analysts' estimates are a better proxy for expected earnings than those from time-series models, and in accordance with the research of Lee et al. (1999)[11]. On the other side Bradshaw et al. (2012) found that only for large, mature and stable firms, over relatively short horizons, analysts' forecasts consistently outperform forecasts from time-series models[12]. Aware of its potential limits, we advocate the importance of a simple technology - for a factor model that determines a full valuation - to have the highest possible control on the input variables and to limit the errors stemming from extrapolation. We cannot but totally agree with the suggestion of Penman (2010): "In valuation, as with most technologies, there is always a tradeoff between simple approaches that ignore some pertinent features and more elaborate techniques that accommodate complexities"[13], and with the thought of Greenwald (2001): "Adding inaccurate to accurate information produces inaccurate information"[14].

The residual income model describes the fundamental value as the sum of two components: book value and discounted residual earnings. Residual earnings are simply the sum of future net income less a charge for shareholders' opportunity cost borne to generate that income, identified by the cost of equity (coe).

$$V_t = BV_t + \sum_{i=1}^{\infty} \left(\frac{RI_{t+i}}{(1+coe_t)^i}\right), \quad RI_{t+i} = E_t\,[NI_{t+i} - (coe_t \times BV_{t+i-1})], \quad coe = r_f + \beta\,(ERP)$$

Alternatively, residual incomes can be expressed as the present value of extra-returns on shareholder capital (expressed by the book value), over the return expected by the investor.

$$V_t = BV_t + \sum_{i=1}^{\infty} \left(\frac{E_t\,[\,(ROE_{t+i} - coe_t) \times BV_{t+i-1}]}{(1+coe_t)^i}\right)$$

As a further clarification, the residual income model represents the "equity side" variant of the probably

[6] Pinto J. E., Henry E., Robinson T. R., Stowe J. D. (2015). *Equity asset valuation. 3rd edition. Hoboken: Wiley.*

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[14] Greenwald B. C. N., Kahn J., Sonkin P. D., van Biema M. (2001). *Value investing: from Graham to Buffett and beyond, Hoboken: Wiley.*

better-known model Economic Value Added® [15]. The EVA® measures the excess operating return compared to the expected return on the capital invested in the business, identified by the weighted average cost of capital (wacc) [16].

$$EV_t = IC_t + \sum_{i=1}^{\infty} \left(\frac{EVA^{®}_{t+i}}{(1+wacc_t)^{\,i}}\right), \quad EVA^{®}_{t+i} = E_t\left[NOPAT_{t+i} - (wacc_t \times IC_{t+i-1})\right],$$

$$EV_t = IC_t + \sum_{i=1}^{\infty} \left(\frac{(ROIC_{t+i} - wacc_t) \times IC_{t+i-1}}{(1+wacc_t)^{\,i}}\right)$$

3. Historical literature context

An earnings-based valuation technique may not seem the best choice on a global level considering that accounting standards differ internationally but, as showed by Frankel and Lee (1998), a simple residual income model without any adjustment accounted for 70% of the cross sectional-variation of stock prices among 20 countries, predicting abnormal returns [17]. As previously recalled, extensive studies have been done on the residual income model as a relevant valuation tool, but fewer have tested it as a basis for asset allocation, especially in the last 10 years notwithstanding the attention that has been paid to several factors that could explain persistent return anomalies. The present work passes through three main areas of financial literature; the market efficiency, the factor theory and the relevance of sell-side analysts forecasts.

The classical notion of market efficiency (weak, semi-strong and strong) initiated by Fama (1970) has been overcome by the notion of near-efficiency presented by Grossman and Stiglitz (1980), which is consistent with the multifactor approach (APT) [18] developed by Ross (1976) [19]. In their framework factors depict the risks that investors cannot eliminate through arbitrage and therefore require a compensation. However, the rational explanation of factors extra-return (a compensation for losses during bad times) is not the only one, since behavioral alternatives have been extensively provided by Shiller (1981), Barberis, Huang (2001) and Thaler, Barberis (2002).

We simply do not rule out the existence of inefficiencies in the market and remain confident in the good sense of investing time and appraisal effort to exploit them. Even if we do not see the price as the best estimate of value all the times, we expect a long-term convergence (as in a co-integrated system) [20]. Besides, in such an environment, a good valuation model should provide better estimates of intrinsic value and give to the user the possibility to earn abnormal returns. If markets are not strictly efficient then it is possible to earn returns that may not be explained by added risks and if it is possible to exploit such anomalies with a residual income model, we infer that the model is a good tool for intrinsic value evaluation. In the present study we will not investigate the source of return of a RIM-based portfolio allocation and we will leave open the question if these returns depend on other specific risk factors or on market inefficiencies determined by investors biases.

For the time being, we will just compare the return of portfolios constructed with RIM with the returns and volatilities of other main factor styles, within a full set of other statistical data. According to Ang (2004) factors are investment styles which deliver high returns over the long run but do not come for free because can underperform in the short run (during "bad times"). Several factors risk premiums have been taken into consideration in academics and among practitioners like the "Value Premium", "Momentum Premium", "Illiquidity Premium", "Volatility premium", "Profitability Premium", but we will focus only on the first one in this study. It is worth pointing out that, while traditional factors have been constructed through market multiples and accounting ratios, the asset management industry is increasingly focusing on quantitative strategies that are much more "data intense" and driven by sophisticated algorithms (artifi-

[15] Economic Value Added® (EVA®) is a service mark of Stern Value Management, formerly Stern Stewart & Co.

[16] BV (book value), RI (residual Income), NI (net income), COE (cost of equity), R_f (risk free rate), ERP (equity risk premium), ROE (return on equity), NOPAT = EBIT * (1-Tax %), IC (invested capital), WACC (weighted average cost of capital), ROIC (return on invested capital).

$$\beta = Beta = \frac{Cov\,(r_i, r_{mkt})}{\sigma^2_{mkt}} = \frac{\sigma_i * \sigma_{mkt} * \rho_{i,mkt}}{\sigma^2_{mkt}} = \frac{\sigma_i}{\sigma_{mkt}} * \rho_{i,mkt}$$

[17] Frankel R., Lee C. M. C. (1998). Accounting valuation, market expectation, and cross-sectional stock returns, *Journal of Accounting and Economics*, 25(3), pp. 283-319.

[18] $r_i = b_{i0} + b_{i1}F_1 + b_{i2}F_2 + \cdots + b_{ik}F_k + \varepsilon_i \qquad r_i = \alpha_i + b_i^{T}\bar{F} + \varepsilon_i$

[19] Ang A. (2014). *Asset management: a systematic approach to factor investing, Oxford: Oxford University Press, pp. 209-211.*

[20] Lee C. M. C., Myers J., and Swaminathan B. (1999). What is the intrinsic value of the Dow? *Journal of Finance, 54(5), pp. 1693-1741.*

cially intelligence based). The definition of factors is constantly changing and some of them still lack a commonly accepted definition, e.g. the "quality factor" as outlined in a recent paper by Hsu, Kalesnik and Kose (2019)[21].

The "Value Premium" has been historically studied with stock market multiples and despite the recent advances in factors' studies, the same approach is employed in the majority of institutional asset allocation strategies. Our idea is to test if the RIM can be a better tool to identify the value factor. According to Fama and French (1998) value stocks outperformed growth stocks in the period 1975-1995 and beat the return of the MSCI index by 3% to 5% yearly depending on the multiple used for the screening. In particular, stocks selection based on value produced a yearly extra-return of 5.09% through BV/P multiple, 4.07% through E/P multiple, 3.92% through C/P multiple and 3.09% through D/P[22]. Nevertheless, Fama and French do not clear the reasons why value deliver a premium, they just show that so it happens. One of the most relevant explanation (on the rationalist side) has been given by Zhang (2005), who addresses the production technology as the justification of a premium. Supposing that value stocks hold a capital that is less productive than growth stocks, their ability to adjust the stock of capital to an external shock is consequently lower[23]. Besides it remains open the interpretation of the behavioral side which explains the value premium as a pure mispricing or, differently said, a valuation mistake of the market. Ali, Hwang and Trombley (2003) e.g. show that risk factors are not responsible for abnormal returns earned by a V/P based on a residual income valuation and that the outperformance seems consistent with the mispricing explanation[24]. We are not trying to answer to any of these hard dilemmas but, aware that a value premium exists, we want to see if a full valuation model can deliver a better performance compared to market multiples. Therefore, we will look if a spread in a portfolio - with stocks ranked through RIM - will produce higher returns and lower volatilities compared to standard factor testing through multiples. Value portfolios are constructed ranking multiples from the lowest, if e.g. we consider P/E (the cheapest), or from the highest, if e.g. we consider the reciprocal E/P (still the cheapest), although we are aware that this is a simplification of reality. It worked in the past, but we know that buying companies with low multiples can expose to a "value trap" as a low P/E could be justified either by a low growth, either by a growth that is risky[25].

$$\frac{P_t}{E_{t+1}} = \frac{payout_t}{coe_t - g_t} = \frac{1}{coe_t} + \frac{(ROE_t - coe_t)}{coe_t} \times \frac{(1 - payout_t)}{(coe_t - g_t)}, \quad g_t = ROE_t \times (1 - payout_t)$$

In the explicit equation, the first term represents the value of the company without growth, the second the return over the cost of capital, the third the present value of all reinvestments. It seems clear that a company with a return inferior to its cost of capital, with low growth, with low value of reinvestment or with high perception of risk on future earnings, or future growth, deserves a low P/E without signaling a mispricing. We find easier to spot such mispricing using a RIM valuation since the relation between input variables and output looks clearer than a P/E valuation.

Finally, since we use analysts estimates as an input of our RIM, it is worth recalling the literature around the relevance of sell-side estimates. While there is some agreement in academics on the relevance of analyst estimates that refer to earnings compared to time-series models, there is more opacity on the relevance of analysts' recommendations. The majority of the studies suggest that there is not a straight relationship between the quality of income estimates and the valuations leading to target prices formation. Bradshaw et al. (2013) say analysts have few incentives to set accurate price targets, which would presumably exhibit little or no predictive ability for future stock returns[26]. Lee et al. (2004) show that analysts generally prefer "glamour" stocks to "value" stocks. They find that stocks receiving more favorable recommendations tend to show positive price momentum, higher trading volume (turnover), higher past and projected growth, more positive accounting accruals and more aggressive capital expenditures[27]. Several studies underline also that valuation heuristics (e.g. multiples heuristic and rela-

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[21] Hsu J., Kalesnik V., Kose E. (2019). What Is quality? *Financial Analyst Journal, 75(2), pp. 44-61.*

[22] Fama E. F., French K. R. (1998). Value versus growth: the international evidence, *Journal of finance, 53(6), pp. 1975-1999.*

[23] Zhang L. (2005). The value premium, *Journal of Finance*, 60(1), pp. 67-103.

[24] Ali A., Hwang L., Trombley M. (2003). Residual-Income-Based Valuation Predicts Future Stock Returns: Evidence on Mispricing versus Risk Explanations, *The Accounting Review*, 78(2), pp. 377-396.

[25] Penman S. H., Reggiani F. (2018). Fundamentals of Value versus Growth Investing and an Explanation for the Value Trap, *Financial Analysts Journal*, 74(4), pp. 103-119.

[26] Bradshaw M. T., Brown L. D., Huang K. (2013). Do Sell-Side Analysts Exhibit Differential Target Price Forecasting Ability? *Review of Accounting Studies*, 18(4), pp. 930-955.

[27] Jegadeesh N., Kim J., Krische S., Lee C. M. C. (2004). Analyzing

tive valuations) are often preferred to formal valuation technique, because the communication from the analysts to the traders/investors can be easier. Damodaran (2005) described analysts' use of multiples as "a story telling experience", where analysts with better and more believable stories are given credit for better valuations. Two other studies, Bradshaw (2004) and Gleason et al. (2012), come to the conclusion that the investment value of analysts' recommendations is reduced substantially when those price targets are formed through valuation heuristic. In particular Bradshaw (2004) concludes that investors would earn higher returns over a one-year holding period by relying on formal DCF/RIM models, that incorporate analysts' consensus earnings forecasts, rather than on analysts' consensus Buy/Sell recommendations alone [28] and Gleason et al. (2012) document that substantial improvements in price target quality occur when analysts appear to be using a residual-income valuation technique rather than a PEG valuation heuristic [29].

4. Data and Portfolio construction

The period considered in our analysis starts on 29 December 1995 and terminates on 29 December 2018, with a dataset that comprises the first 600 companies by market capitalization in United States (US) and Western Europe (not limited to the Eurozone). We opted for this number of companies to compare our results to the main domestic indexes that will represent our benchmarks for market return: the S&P 500 for US and the STOXX 600 for Europe. The period 1995-2018 has been chosen because analysts' estimates are less numerous before 1995 and in order to compare different sub-periods that included major market corrections and bull markets. All periods start and end on 29 December and in addition we checked 4 sub-periods: 1995-2005, which includes the "dot.com" bubble; 2000-2010, which includes the financial crisis of 2007-2009; 2007-2010, to test factors response during full market downturn; 2010-2018, which represents one of the longest bull market in history.

The study required a wide range of financial data, collected from FactSet Research Systems and Bocconi University databases, and in particular analysts' estimates, used both for the implementation of our RIM model and for market multiples. Since forecasts are essential to our study, we applied a window of 60 days on analysts' estimates data to exclude form our sample older and virtually not updated estimates. Regarding financial reporting data we applied a time lag of 90 days to avoid the "look-ahead bias" and to use only information available at the time of the trade. The assumptions on which our RIM model is based (underlying the V/P ratio) will be discussed in detail in the next paragraph and we are now quickly specifying the market multiples used as factors: analysts' target price/price (TP/P), represents the average target price of analysts (within a 60 days consensus window), at the time of the monthly valuation, divided by the price of the stock (the price refers to the rebalancing day, so at the end of every month); $ROE^{(trailing)}$ represents the 5 year average of net $income_{(t)}$ divided by average book value $[(BV_{(t)}+BV_{(t-1)})/2]$; $ROE^{(forward)}$ represents the $EPS_{(t+1)}$ estimated by analysts (first unreported fiscal year) divided by $BV_{(t)}$ (last reported year); $P/E^{(trailing)}$ represents the price divided by the last reported EPS (with 90 days' time-lag); $P/E^{(forward)}$ represents the price divided by the $EPS_{(t+1)}$ estimated by analysts; $PEG^{(trailing)}$ represents the $P/E^{(trailing)}$ divided by average expected growth from t+1 to t+3; $P/E^{(forward)}$ represents the $P/E^{(forward)}$ divided by average expected growth from t+1 to t+3; $P/BV^{(trailing)}$ represents the price divided by the last reported book value per share (with 90 days' time-lag); $P/BV^{(forward)}$ represents the price divided by the $BV_{(t+1)}$ obtained through the "clean surplus accounting" relationship $[BVPS_{(t+1)} = BVPS_{(t)} + EPS_{(t+1)} - DPS_{(t+1)}]$; $EV/EBITDA^{(historical)}$ represents the enterprise value (with 90 days' time-lag) divided by the last reported EBITDA; $EV/EBITDA^{(forward)}$ represents the enterprise value divided by the $EBITDA_{(t+1)}$ estimated by analysts, Size represents the market value of the company.

Afterwards we used FactSet's integrated tools for quantitative research (Alpha Testing application) [30] to test the ability of our RIM model and other factors to forecast future returns. Both for the full period and the sub-periods we constructed equally-weighted monthly portfolios according to factors rank, expecting a higher return in the highest part of the ranking. The ranking follows a descending order or an increasing order depending on the type of multiple: descending in case of V/P, TP/P and ROE (the higher the multiple the higher the expected return); increasing in case of P/E, PEG, P/B, EV/EBITDA and Size (the lower the multiple the higher the expected return). We divided the universe of available securities in quintiles according to the factor rankings and we added two further

the analysts: When do recommendations add value? *Journal of Finance*, 59(3), 1083-1124.

[28] Bradshaw M. T. (2004). How Do Analysts Use Their Earnings Forecasts in Generating Stock Recommendations? *The Accounting Review*, 79(1), pp. 25-50.

[29] Gleason C. A., Johnson W., Li H. (2012). The Earnings Forecast Accuracy, Valuation Model Use and Price Target Performance of Sell-Side Equity Analysts, *Contemporary Accounting Research*, 30(1), pp. 80-115.

[30] The Alpha Testing application in FactSet is used to build models specifying the factors to test and customizing fractile assignments.

scenarios only for the RIM valuation (V/P ratio): "top 20/bottom 20" and "top 30/bottom 30" stocks. The portfolios are rebalanced every month if any changes in ranking occur and the following data are analyzed for all factors: spread yearly returns (long "1 quintile" – short "5 quintile", or long "top 20 or 30" – short "bottom 20 or 30"), yearly returns, cumulative returns, Sharpe ratios[31], information ratios[32], alphas[33], betas[34], information coefficients[35], portfolios turnover, maximum drawdowns and spread return correlations[36]. Among the listed metrics we want to clarify just the relevance of the information coefficients in our analysis. The IC represents the correlation between the actual values of a forecasted variable and its predicted returns, namely an IC equal to one indicates perfect forecasting skill whereas an IC equal to zero indicates no forecasting skill. The IC represents a Spearman's rank correlation coefficient[37] that is a nonparametric test which measures the strength and direction of association between two variables that are measured on an ordinal or continuous scale. The Spearman rank IC is essentially the Pearson correlation coefficient between the ranked factor scores and ranked forward returns and it is a useful test when Pearson's correlation cannot be run due to violations of normality, a non-linear relationship or when ordinal variables are being used. To establish the forecasting skills of our selected metrics we investigate if the contribution to alpha really comes from the ranking within every factor's portfolio. Grinold's (1989) fundamental law of active management states that $IR \approx IC \times \sqrt{BR}$ where IR is the information ratio, IC the information coefficient and BR is the breadth of the strategy[38]. A strong assumption implied in the previous formula is the absence of constraints on portfolio construction, with positions that can be long or short and of any size. Clarke, de Silva and Thorley (2002) introduced a scaling factor called "transfer coefficient" (TC < 1) so that $IR \approx IC \times TC \times \sqrt{BR}$, underling the potential value lost due to constraints on portfolio size and turnover. For simplicity, in the following example we overlooked the impact of TC. When hundreds of stocks can be traded (high breadth) even a low IC can generate profitable strategies; as an example, if 200 independent trades are executed in one year it is possible to generate an IR of 0.50 with an IC of 3.5% $(0.5 \approx 0.035\, x \sqrt{200})$. It is worth to note that a crucial assumption is the independence of forecasts and therefore it may be hard to correctly define breadth, as investment decisions tend to be correlated. If we are buying 100 stocks for 100 different reasons, we are making 100 different bets, while if we are buying 100 stock because they all have a low multiple, we are making one big bet on a specific factor, not 100. According to Grinold and Kahn (2000)[39], if information ratios have a normal distribution, a "good" investment strategy can be identified within the top quartile of the population. Therefore, a "good" IR can be assumed to be greater or equal to 0.5. This implies that if we made 12 forecasts in one year (one per month, considering high correlations between stocks traded within the same month) we would need an information coefficient (IC) of 14% to obtain a "good" performance. As a rule of the thumb, portfolio managers would view an IC of 5% as "good", an IC between 10% and 20% as "very good" and one above 20% as "extremely good".

4.1. RIM-based V/P multiple methodology

The structures of the model we used to implement a full valuation of stocks in US and Europe and generate portfolio rankings according to a RIM-based V/P, have been essentially two, one with a truncation at the third year of analysts' estimates and one with additional growth of 5 years. The first model relies on analysts' estimates till year 3, as previously stated, because after that date we lack enough observations.

$$V_t = BVPS_{y_0} + \frac{EPS_{y_1} - (coe_t \times BVPS_{y_0})}{(1+coe_t)^{\frac{(y_1-t)}{(30\times12)}}} + \frac{EPS_{y_2} - (coe_t \times BVPS_{y_1})}{(1+coe_t)^{\frac{(y_2-t)}{(30\times12)}}} + \frac{EPS_{y_3} - (coe_t \times BVPS_{y_2})}{(1+coe_t)^{\frac{(y_3-t)}{(30\times12)}} \times (coe_t - g_t)}$$

31 $Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p}$

32 $Information\ ratio = IR = \frac{\alpha}{\bar{\sigma}} = \frac{R_p - R_{bmk}}{\sqrt{var(R_p - R_{bmk})}}$

33 $\alpha_p = R_p - \beta_p(R_{bmk})$

34 $\beta = Beta = \frac{Cov\ (r_p, r_{bmk})}{\sigma_{bmk}^2} = \frac{\sigma_p \times \sigma_{bmk} \times \rho_{p,bmk}}{\sigma_{bmk}^2} = \frac{\sigma_p}{\sigma_{bmk}} \times \rho_{p,bmk}$

35 $IC \approx \frac{IR}{\sqrt{BR}}$, IC (information coefficient) = correlation of the manager's forecast with the actual returns, BR = strategy's breadth

36 $F_1 - F_n\ return\ correlation = \frac{Cov\ (spread_i,\ spread_j)}{\sigma_{spread_i} \times \sigma_{spread_j}}, s.t.\ spread_i = F_1^i - F_n^i, spread_j = F_1^j - F_n^j, = \frac{Cov\ (F_1^i - F_n^i,\ F_1^j - F_n^j)}{\sigma_{F_1^i - F_n^i} \times \sigma_{F_1^j - F_n^j}}$

37 $r_s = \rho_{rg_X, rg_Y} = \frac{\mathrm{cov}\ (rg_X, rg_Y)}{\sigma_{rg_X} \times \sigma_{rg_Y}}$, where X_i, Y_i are converted to ranks rgX_i, rgY_i

38 Ang A. (2014). *Asset management: a systematic approach to factor investing*, Oxford: Oxford University Press, pp. 310-311.

39 Grinold R. C., Kahn R. N. (2000). *Active Portfolio Management*, New York: McCraw-Hill.

The EPS_{y1} represents the analysts' estimate for the first unreported fiscal year, not the 12-month forward estimate from the date of backtest/valuation. Therefore, if the date of valuation occurs in September $year_t$ and the EPS_{y1} estimate pertains to the EPS published in March $year_{t+1}$, the interest used for discounting will be converted to a semi-annual rate. We took into account all these timing differences through an appropriate discounting, proportional to the months of distance between the date of backtest and the date of reference of the analysts' estimate. Besides, as further clarification, if the company reports its full year results in March $year_t$, the EPS_{y1} at February $year_t$ still refers to the fiscal $year_{t-1}$ and we convert the discount factor to a monthly rate. We assumed for simplicity that the "clean surplus relationship" held in our period of analysis, even if shares transactions (e.g. buybacks) or certain changes in fair value could have had an impact on the overall results, especially in the last years. While these issues could necessitate a further and deeper analysis in a future research, the approach to tackle them should be slightly different from the present one. Considering that it will not be easy to find analysts' estimates related to items included in the comprehensive income, a forecasting function based on historical data should be embedded in pairs with analysts' estimates to obtain the expected BVPS. The forward $BVPS_{y1}$ has been calculated through the "clean surplus accounting" relationship ($BVPS_{y1} = BVPS_{y0} + EPS_{y1} - DPS_{y1}$) using analysts' estimates on earnings and dividends. When estimates on EPS and DPS - for the three fundamental years to implement the model - were missing in our databases, we made some common sense adjustments. If an estimate for EPS_{y1} is missing the company will be rejected and will go into "n/a" portfolio, if EPS_{y2} is missing we will consider it to be equal to EPS_{y1} (considering no growth for the second forward fiscal year), if EPS_{y3} is missing we will multiply EPS_{y2} for the EPS growth of the previous year reduced by 1/3. Furthermore, in case any DPS estimate miss we will replace it with the last 5 years reported payout ratio multiplied by the estimated forward EPS. Regarding the discounting process, we estimated the cost of equity capital through CAPM (with time-varying risk-free rate and equity risk premium), contrary to the majority of previous studies which employed the Fama-French industry cost of capital or fixed rates (as showed in table 1). Abarbanell and Bernard (1995) and Frankel and Lee (1998) found that the choice of r_e had a small impact on their cross-sectional analyses, while it was important to incorporate time-varying rates[40].

Table 1 – Cost of equity capital estimates in previous studies employing RIM-based valuations

	Frankel, Lee (1998)	Dechow et al. (1999)	Lee et al. (1999)	Gode, Mohanram (2001)	Ali et al. (2003)	Bradshaw (2004)	Gleason et al. (2012)
Discount Rate	Fama-French 3 factors, by industry	12%	Time-varying riskless rate; Fama-French 1 and 3 factors risk premia, by industry	Fama-French 3 factors, by industry	Time-varying riskless rate; Fama-French 3 factors risk premia, by industry	Time-varying riskless rate; Fama-French 3 factors risk premia, by industry	Industry disc. rates; Fixed at 8%, 12%, 16%; CAPM

Source: Cited papers in bibliography, in particular: Ali A., Hwang L., Trombley M. (2003). Residual-Income-Based Valuation Predicts Future Stock Returns: Evidence on Mispricing versus Risk Explanations, *The Accounting Review*, 78(2), pp. 377-396.

The CAPM has been criticized both on its theoretical foundations and due to various empirical anomalies (e.g. Fama-French three-factor and five-factor model). However, it is still conventionally considered the model of reference to estimate the cost of equity by the business valuer community[41]. Although the simplicity of the model has overshadowed its inaccuracies, it should be acknowledged for incorporating the two main risks faced by a company: operating leverage and financial leverage. First of all, in our monthly CAPM estimate [$coe = r_f + \beta\ (ERP)$], we calculated the risk-free rate as the five-year average yield of 10-year bonds and we did not take into account the yield prevailing at the date of valuation/backtest; in particular the 10-Y T-Bond for United States and the 10-Y German T-Bund for Europe. While the decision for US was an obvious choice, regarding Europe we had to choose between a formally more correct method but longer to implement (selecting for every country its domestic financial metrics) and one less precise but easier to implement and to test (identifying one general metric for all European markets; Germany as a proxy of risk-free rate and EU broad market index to calculate betas and ERPs). We opted for the second option regarded

[40] Lee C. M. C., Myers J., and Swaminathan B. (1999). What is the intrinsic value of the Dow? *Journal of Finance, 54(5), pp. 1693-1741.*

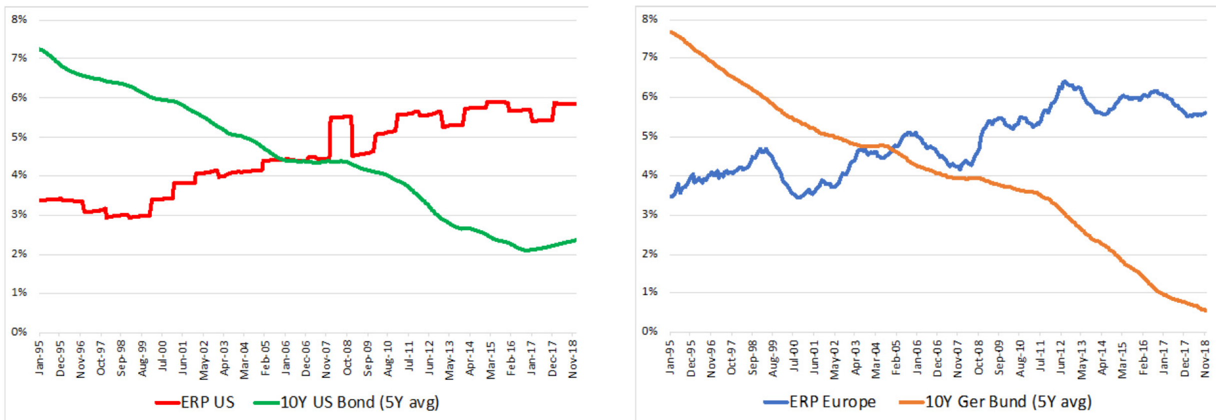
[41] Bini M. (2018). Implied cost of capital: how to calculate it and how to use it, *Business Valuation OIV Journal*, Fall, 0(0), pp. 5-32.

as a good balance between costs and benefits, especially in estimating betas and equity risk premiums. By applying a German riskless rate and a European equity risk premium to the valuation of some European countries, we are not considering every specific country risk directly, but this hidden risk is expected to be recovered through higher betas, explained by higher volatilities with respect to a European market index. We calculated monthly betas between companies and market indexes (the S&P 500 for US and the STOXX 600 for Europe) using 5 years of weekly price returns. Afterwards we adjusted the betas stemming from regressions through the Blume technique [*Beta adjusted = 2/3 Raw β + 1/3 x Mkt β*]. Lastly, we calculated the equity risk premiums for US and Europe considering both the implied ERP and an inverse relationship between the ERP and the 10-Y Bond. Basically, we obtained the ERPs through an average of two time-series: the first one formed by the implied ERPs and the second one by ERPs that grows in reverse to a decrease in government bond rates weighted by 50%.

$$ERP_t = 0.5 \times \{Implied\ ERP_t + [ERP_{t-1} \times 0.5 \times (-\Delta\ 10Y\ Bond_{t-1,t})]\}$$

The first ERP_{t-1}, at the time of the first valuation in 1995, has been calculated by making an average of the previous 5 years implied ERP, which came in at 3.5% both in US and Europe. Besides, we calculated the implied ERP by using yearly data provided by Damodaran for US (obtained through FCFE) and we calculated on ourselves data for Europe through a dividend discount model. As an input of the DDM to calculate the implied ERP in Europe we used aggregate dividend estimates on the European stocks, the 10-Y German T-Bund and the expected European long-term GDP growth. The results obtained are showed in graph 1.

Graph 1 – (1995-2018) Sx: US ERP, 10Y US Bond (5Y avg); Dx: EU ERP, 10Y Ger Bund (5Y avg)



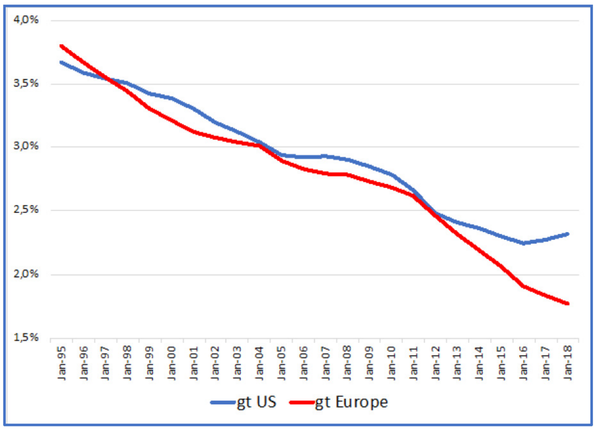
Source: FactSet Research Systems, Damodaran A. "Equity Risk Premiums (ERP)" for US, Bocconi University databases, our estimates

Finally, we calculated the long-term growth for both US and Europe looking at the compounded GDP growth (constant prices) for developed economies between 1980 and 2017 (resulted in 2.3%) and the 10-Y respective government yield at the date of monthly backtest, applying the following proportion:

$$g_t = 0.7 \times (g_{long\ term}) + 0.3 \times (10Y\ Bond),$$
$$where\ g_{long\ term} = US\ GDP(constant\ prices)_{1980-2017} = 2.3\%$$

The long-term growth (g_t) implied in the terminal value calculation (TV) has been the same for every stock as in the long term every company should grow in line with the general economy. The results obtained for US and European g_t are shown in graph 2:

Graph 2 – (1995-2018) Long-term growth estimated for TV calculation in US and Europe



Source: FactSet Research Systems, Bocconi University databases our estimates

The second model of RIM studied uses the same inputs of the first one but implies a further forecasting horizon of 5 years, for a total of 8 years of estimates. We did not use analyst estimates for the further 5 years but the sustainable growth [g_s = (1-payout) x ROE] calculated as the last 5 years reported average. In order to calculate the book values after the first forecasting period we used the 5 years average dividends payout ratio.

$$V_t = BVPS_{y_0} + \frac{Residual\ EPS_{y_1}}{(1+coe_t)^{\frac{(y_1-t)}{(30\times12)}}} + \frac{Residual\ EPS_{y_2}}{(1+coe_t)^{\frac{(y_2-t)}{(30\times12)}}} + \frac{Residual\ EPS_{y_3}}{(1+coe_t)^{\frac{(y_3-t)}{(30\times12)}}} + \frac{[EPS_{y_3}\times(1+g_s)^1]-(coe_t\times BVPS_{y_3})}{(1+coe_t)^{\frac{(y_4-t)}{(30\times12)}}} +$$
$$\frac{[EPS_{y_3}\times(1+g_s)^2]-(coe_t\times BVPS_{y_4})}{(1+coe_t)^{\frac{(y_5-t)}{(30\times12)}}} + \frac{[EPS_{y_3}\times(1+g_s)^3]-(coe_t\times BVPS_{y_5})}{(1+coe_t)^{\frac{(y_6-t)}{(30\times12)}}} + \frac{[EPS_{y_3}\times(1+g_s)^4]-(coe_t\times BVPS_{y_6})}{(1+coe_t)^{\frac{(y_7-t)}{(30\times12)}}} +$$
$$\frac{[EPS_{y_3}\times(1+g_s)^5]-(coe_t\times BVPS_{y_7})}{(1+coe_t)^{\frac{(y_8-t)}{(30\times12)}}\times(coe_t-g_t)},$$

5. Results and performance of RIM

Our research highlights that a RIM-based V/P ratio, based on previous assumptions, forecasts abnormal returns. In particular we show its ability to outperform main market indexes (both in US and Europe), other factor styles and analysts' recommendations by using their own estimates of financial reporting data. It is worth remembering that all returns presented in the following sections includes dividends (i.e. total return) and are gross of financial transactions costs and taxes on capital gains. Nevertheless, taking into account the turnover of our top quintiles portfolios that ranged from a minimum of 6% and a maximum of 40% monthly turnover (looking at US and Europe together), we estimated that the yearly cost for brokerage fees can range between 0.2% and 0.7% yearly (through various combinations of discount brokers and institutional brokers fees structures). These additional costs do not diminish the results of the study, even though it is consistent to account them for portfolios showing high turnover. With respect to taxes the analysis proves to be more complicated as we should apply a financial tax once the capital gain is realized, at the end of each fiscal year or potentially every month, depending on the domicile and the structure of the investor. Furthermore, we do not take into account the liquidity issue as we do not consider the implicit cost determined by the price movement against a trade with significant volume (price impact). A recent paper by Li, Chow, Pickard and Garg (2019) shed light on the matter, pointing out the potential impact of transaction costs on factor-investing strategies[42]. They show that the price impact is predictable because it is directly related to the security's liquidity and the size of the trade. In particular, they explain that a fund incurs approximately 30 bps of trading costs as a result of market impact for every 10% of a stock's average daily volume traded in ag-

[42] Li F., Chow T-M., Pickard A., CFA, Garg Y., CFA (2019). Transaction Costs of Factor-Investing Strategies, *Financial Analysts Journal*, 75(2), pp. 62-78.

gregate by the funds tracking a factor-investing index. Considering several US factor-investing strategies from 1968 to 2016, they show that with $10 billion in AUM the annual market impact cost can range from 0.10% to 2.7% and in detail, that the "fundamental value" strategy endures an annual market impact cost of 0.28% within an average 25% portfolio turnover. Since our aim is not to propose a trading strategy but to test the soundness of RIM as a valuation tool and potentially as a risk indicator, we will not speculate on the impact of trading costs (both explicit and implicit) and taxes. We believe that all these costs do not undermine the soundness of our analysis concerning the potential of RIM in producing better empirical estimates of value. Again, it is just worth noticing that factoring in all costs associated to an investment strategy makes always extremely hard to beat the market in real life since markets are nearly efficient.

5.1. Forecasting excess return

As we are going to summarize, all RIM-based V/P top ranked portfolios outperformed local markets' indexes, producing statistically significant alphas, both in US and Europe.

5.1.1. United States

We start the analysis introducing the results obtained in the US market where the model has been extensively studied in financial history. It is worth to point that, from now on, the expression V/P will be furtherly specified as "V/P" and "V/P (+5Y)", to better outline the two versions of the model developed in chapter 4.1. Both the RIM-based V/P with truncation at the third year of estimates "V/P" and the RIM-based V/P with further 5 years of estimates "V/P (+5Y)" top portfolios outperformed the main US market index. As shown in Table 2 all first ranked portfolios (the ones with the higher V/P) outperformed the index, posting a yearly extra-return between 4% and 6.5%, depending on the type of multiple considered, in the period 1995-2018. The monthly alphas produced by RIM-based allocation came in between 0.34% and 0.57%, with a statistical significance above 95% (details of t-stat for all portfolios at APPENDIX A, Table - A2). It can be noticed at first glance that the more complex model "V/P (+5Y)" does not beat the simpler model "V/P", which takes into account very few assumptions and data and just uses three years of analysts' forecasts. Additionally, the simpler model should be more conservative a priori: in "V/P (+5Y)" we are extrapolating the company's past 5 years sustainable growth and projecting it into the future while this growth should be greater than the growth of the economy used in terminal value calculation. First note: a simpler and more conservative model seems to perform better. Long-short portfolios yearly returns enrich the first analysis as the "V/P" produces yearly self-financing returns ranging from +7.6% to +12.6%. This fact signals an extremely good model at detecting not only undervalued companies but especially overvalued ones, placing them in the last quintile of the ranking. Nevertheless, it is essential to check if these returns are driven by higher volatilities and/or higher betas. As shown with all details in APPENDIX A (Table - A1, A2, A3), the RIM-based V/P reported a Sharpe ratio ranging from 0.75 to 0.84 (S&P 500 Sharpe 0.58), an information ratio ranging from 0.43 to 0.54, a beta lower than the market ranging from 0.91 to 0.98 (calculated with monthly portfolio returns). We checked the correlation between the actual values of the forecasted returns and their predicted values through IC coefficients (APPENDIX A, Table - A5), noticing that, while the simpler "V/P" shows always a positive and significant correlation, the "V/P (+5Y)" shows a positive (but low) correlation and only for portfolios organized through quintiles and not for "Top20" and "Top30"stocks.

The "V/P" shows instead an IC of 5% even with a forecasting horizon of one month and the information coefficient grows from 12% to 20% as the forecasting horizon is moved ahead. As explained in the first part of this study, an IC between 10% and 20% is considered as "very good" by the investment community and we already obtained such values with a forecasting horizon from six (IC range 12%-14%) to twelve months (IC range 15%-16%). Having shown the forecasting skills of our selected metrics we can infer that the contribution to alpha really comes from the ranking within "V/P" factor's portfolios. Finally, we observed the performance of our metrics in the sub-periods within 1995-2018 (details in APPENDIX C): both "V/P" and "V/P (+5Y)" top ranked long-only portfolios outperformed the market index across periods 1995-2005, 2000-2010 and 2010-2018 and slightly underperformed during the crisis 2007-2010. Surprisingly, "V/P" based long-short portfolios registered a positive performance across all times, with two minor exceptions (details in APPENDIX C). It is worth to pay further attention to the most recent period, known as one of the longest bull markets in history. Even in the period 2010-2018 the RIM-based valuation has been able to produce returns higher than the market, in particular the long only "V/P" portfolios beat the S&P500 by around 3% and the long-short "V/P" portfolios posted returns between 1.7% and 3.8%. The last results are surprising regarding to the fact that the last years were considered to be negative for value portfolios, constantly outperformed by growth portfolios. For the time being we can just signal that this depends also on how we define a value portfolio, as low multiples are not always a good proxy of value.

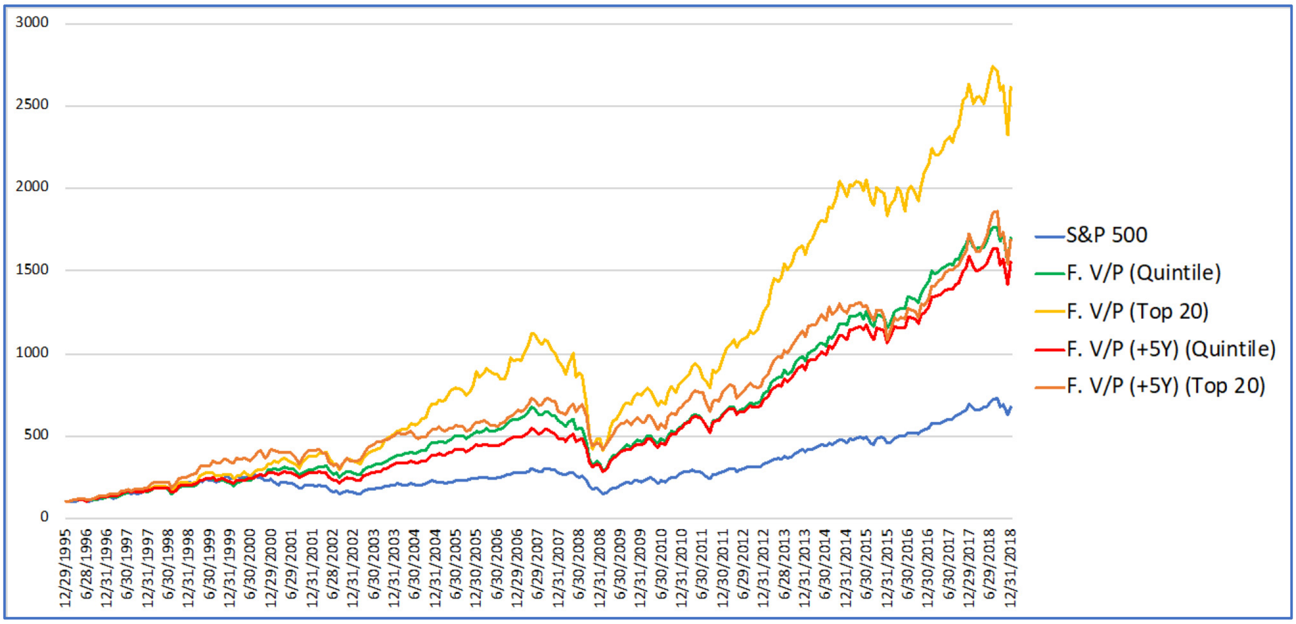
Table 2 – (1995-2018) US - Yearly compounded returns of monthly rebalanced portfolios: long-short strategy (F1-FN), long only strategy (quintiles, top/bottom 20, top/bottom 30), S&P 500

	Long-short yearly spread	Universe yearly return					Benchmark yearly return
Factor	F1-FN	1	2	3	4	5	S&P 500
F. V/P (Quintile)	7,65	13,05	12,18	12,38	7,63	2,89	8,66
F. V/P (Top 20)	12,61	15,18	10,09	-0,79			8,66
F. V/P (Top 30)	9,47	15,11	10,02	2,44			8,66
F. V/P (+5Y) (Quintile)	5,43	12,62	11,93	12,06	7,44	4,50	8,66
F. V/P (+5Y) (Top 20)	6,86	13,02	10,07	3,06			8,66
F. V/P (+5Y) (Top 30)	5,83	12,57	10,22	3,52			8,66

Source: FactSet Research Systems (Alpha Testing), Bocconi University databases, our estimates

Before passing to the next chapter we show below the graph of the cumulative returns of a selection of RIM-based V/P factors compared to the market index. It can be noticed that long-only strategies are not immune to market deep correction, indeed the maximum drawdown suffered by our V/P portfolios ranged from -43% to -64%. In most of the real-world portfolio management processes, a big absolute loss in a certain unit of time would be considered as unacceptable, limiting the employment of our approach. Consistently with our research purpose (testing the soundness of RIM over a long period of time without constraint) we did not assume a reaction function of the strategy to the absolute losses. On the other side, it is worth pointing out that, in monthly portfolio rebalancing, we only considered relative valuations among companies without placing a threshold to the V/P multiple. There are times in our dataset in which the estimated value is lower than the price (V/P multiple below 1) for most companies, signaling an overvaluation of the broad equity market. We could have included a timing function to rotate from equity to bonds once a certain threshold of V/P is broken. As interesting as it is both on the asset management side and the fundamental valuation side, we limited our analysis to the easiest long-only feasibility.

Graph 3 – (1995-2018) US - Cumulative returns of monthly rebalanced portfolios: Long only V/P, S&P 500



Source: FactSet Research Systems (Alpha Testing), Bocconi University databases, our estimates

5.1.2. Europe

We now present the results obtained in the European market where companies of different countries have been valued with the same assumptions, the same risk-free rate (identified in the German 10Y Bund), the same equity risk premium and betas calculated in comparison to the main market index (STOXX 600). Both the RIM-based V/P with truncation at the third year of estimates "V/P" and the RIM-based V/P with further 5 years of estimates "V/P (+5Y)" top portfolios outperformed the STOXX 600. As shown in Table 3 all first ranked portfolios (the one with the higher V/P) outperformed the index, posting a yearly extra-return between 4.5% and 7.4% depending on the type of multiple considered, in the period 1995-2018. The monthly alphas produced by RIM-based allocation came in between 0.47% and 0.63%, with a statistical significance above 95% (details of t-stat for all portfolios at APPENDIX B, Table - B2). Like in US, it can be noticed at first glance that the more complex model "V/P (+5Y)" does not beat the simpler model "V/P", which takes into account very few assumptions and data, just using three years of analysts' forecasts. We specify again how the simpler model should be more conservative than the "V/P (+5Y)" ratio as we are extrapolating the company's past 5 years sustainable growth and projecting it into the future. Again, a simpler and more conservative model seems to perform better. Long-short portfolios yearly returns enrich the analysis as the "V/P" produces yearly self-financing returns ranging from +4.1% to +4.6%. RIM confirms its ability not only in signaling undervalued companies, but also in detecting overvalued ones, placing them in the last quintile of the ranking. Nevertheless, it is essential to check if these returns are driven by higher volatilities and/or higher betas. As shown in detail in APPENDIX B (Table - B1, B2, B3), the RIM-based V/P reported a Sharpe ratio ranging from 0.55 to 0.71 (STOXX 600 Sharpe of 0.27), an information ratio ranging from 0.45 to 0.82, a beta lower than the market ranging from 0.90 to 0.95 (calculated with monthly portfolio returns). Checking the correlation between the actual values of the forecasted returns and its predicted values through IC coefficients (APPENDIX B, Table - B5) we found a different picture compared to US. It has to be noticed that ICs signal a positive and significant correlation between forecasts and returns only with a time horizon of 3 years (IC ranging from 3% to 9% depending on the RIM-based V/P considered). Such a time horizon would be considered as an eternity by the investment industry in a long-only strategy, but at the same time the long-short strategy reported positive and significant results also in Europe. Finally, we observe the performance of our metrics in the sub-periods within 1995-2018 (details in APPENDIX D): both "V/P" and "V/P (+5Y)" top ranked long-only portfolios outperformed the market index across periods 1995-2005, 2000-2010 and 2010-2018 and slightly underperformed during the crisis 2007-2010. It is necessary to pay further attention, also in Europe, to the most recent period, known as one of the longest bull markets in history. Even in the period 2010-2018 the RIM-based valuation has been able to produce returns higher than the market, in particular the long only V/P portfolios beat the STOXX 600 by around 6% and the long-short V/P portfolio posted returns between 1% and 3.4%.

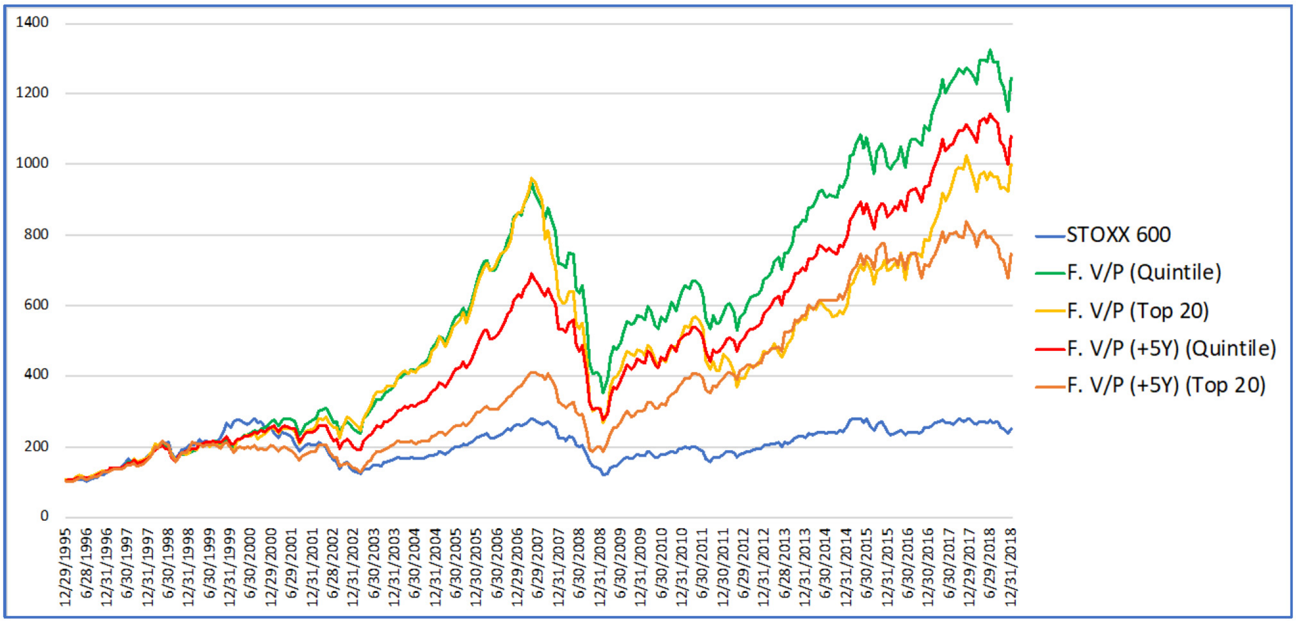
Table 3 – (1995-2018) Europe - Yearly compounded returns of monthly rebalanced portfolios: long-short strategy (F1-FN), long only strategy (quintiles, top/bottom 20, top/bottom 30), STOXX 600

	Long-short yearly spread	Universe yearly return					Benchmark yearly return
Factor	F1-FN	1	2	3	4	5	STOXX 600
F. V/P (Quintile)	4,55	11,54	9,95	8,93	7,66	5,18	4,11
F. V/P (Top 20)	4,59	10,49	8,89	3,60			4,11
F. V/P (Top 30)	4,14	10,64	8,91	4,21			4,11
F. V/P (+5Y) (Quintile)	3,23	10,85	9,87	9,03	7,84	6,09	4,11
F. V/P (+5Y) (Top 20)	2,23	9,10	8,99	4,35			4,11
F. V/P (+5Y) (Top 30)	3,06	9,38	9,09	4,33			4,11

Source: FactSet Research Systems (Alpha Testing), Bocconi University databases, our estimates

We end this paragraph by showing in graph 4 the cumulative returns of a selection of RIM-based V/P factors compared to the STOXX 600. It can be noticed that, as previously underlined in US, also in Europe long-only strategies are not immune to market deep corrections, indeed the maximum drawdown suffered by our V/P portfolios ranged from -55% to -70%.

Graph 4 – (1995-2018) Europe - Cumulative returns of monthly rebalanced portfolios: Long only RIM-based V/P, S&P 500



Source: FactSet Research Systems (Alpha Testing), Bocconi University databases, our estimates

5.2. RIM compared to other factors

The majority of RIM-based V/P top ranked portfolios outperformed other factors considering long-short portfolios and produced higher Sharpe ratios and information ratios, considering long-only portfolios. The other factors that produced the best adjusted returns, taking into account Sharpe ratios and information ratios have been ROE (forward) in United States and ROE (forward) and EV/EBITDA in Europe.

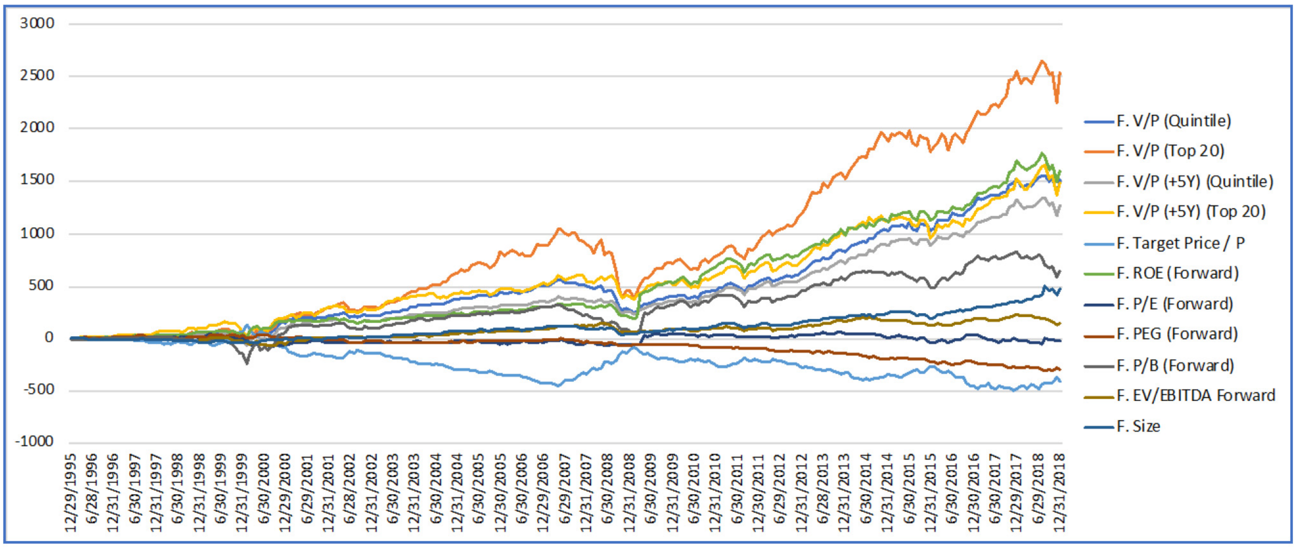
5.2.1. United States

We are now going to observe the results delivered by all factors taken into account. In United States, during the full period of analysis (1995-2018), the top long-only portfolios based on V/P multiples outperformed the majority of other factors considered (TP/P, P/E, PEG, P/B, EV/EBITDA and Size) with the only exception of ROE (Forward), which posted similar returns. Besides, all RIM-based V/P top ranked portfolios reported Sharpe ratios and information ratios higher than other factors, in particular Sharpe ratios ranging from 0.75 to 0.84 and information ratio ranging from 0.43 to 0.54. As previously stated, the best performance among other factors has been shown by ROE (Forward) with a Sharpe ratio of 0.59 and an information ratio of 0.31.

It is interesting to notice that all V/P top ranked portfolios registered a lower beta (ranging from 0.92 to 0.98) compared to the other factors (ranging from 1.03 to 1.47). Moreover, looking at IC coefficients (APPENDIX A, Table - A5), it can be inferred that V/P is the best return forecaster among all factors, posting the highest values either at 6 months, 12 months and 36 months.

We will not focus on all yearly returns' differences (details in APPENDIX A), rather we want to stress the superior returns obtained through a long-short strategy, which is self-financing. Graph 5 shows the cumulative long-short portfolios' performances of all factors (that use forward estimates) analyzed in the period 1995-2018.

Graph 5 – (1995-2018) US - Cumulative returns of monthly rebalanced portfolios: Long-short RIM-based V/P, long-short multiple's factors



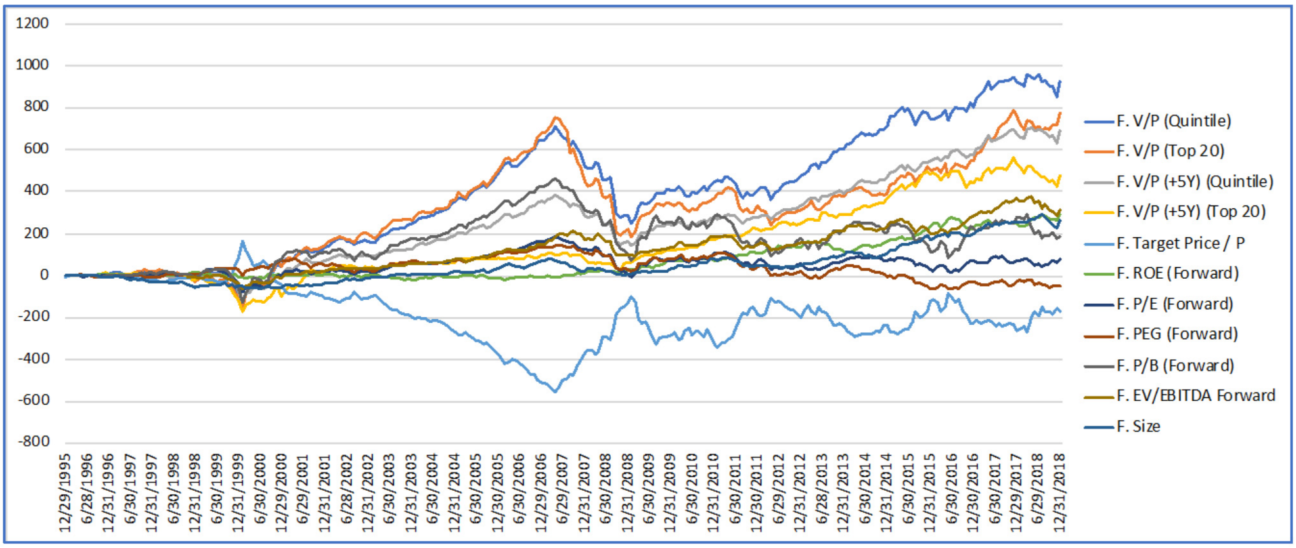
Source: FactSet Research Systems (Alpha Testing), Bocconi University databases, our estimates

At first glance it is surprising to see a significant spread between long-short performances of the V/P (Top 20) multiple compared to the TP/P multiple, which represents analyst recommendations. Since we based our model on analysts' estimates of EPS and DPS we did not expect such a difference, that instead appears consistent with the academic literature on analysts' biases presented in chapter 3. Furthermore, long-short spread returns coming from analysts' recommendations did not show any significant correlation with other valuation models across the full period 1995-2018 (details APPENDIX A, Table - A7, F1-FN return correlations). Analysts did not exhibit differential target price forecasting ability as both the last two quintile portfolios (the least recommended stocks) outperformed the first two quintile portfolios (the most recommended stocks) and through their target prices produced a negative yearly compounded long-short return of - 3.5%. While TP/P multiple produces the worst strategy in our sample for the full period (1995-2018) it is worth to notice that in the last sub-period (2010-2018) it added some value producing a yearly spread long-short return of 2.6%. Nevertheless, we also notice that in the last years almost all factors, among all ranked portfolios, showed positive returns with increased correlations among them. This may be due to the high level of liquidity in financial markets driven by central banks, which may have contributed to the alteration of historical risk premiums among different factors.

5.2.2. Europe

Moving to Europe and considering yearly compounded returns (both long-only and long-short) during the full period of analysis (1995-2018), the top ranked portfolios based on V/P multiples outperformed some of the factors considered [ROE$^{(trailing)}$, PEG$^{(forward)}$,TP/P, P/E, P/B and Size] while producing similar results compared with ROE$^{(forward)}$, PEG$^{(trailing)}$ and EV/EBITDA (both Historical and Forward). Besides, all RIM-based V/P top ranked portfolios reported Sharpe ratios and information ratios higher than other factors [with the exception of ROE$^{(forward)}$ and EV/EBITDA], recording Sharpe ratios ranging from 0.55 to 0.71 and information ratio ranging from 0.45 to 0.82. As previously stated, the best performance among other factors top ranked portfolios has been shown by ROE (Forward) with a Sharpe ratio of 0.64 and an information ratio of 0.86, by PEG (trailing) with a Sharpe ratio of 0.5 and an information ratio of 0.69, by EV/EBITDA (Historical) with a Sharpe ratio of 0.69 and an information ratio of 1.08. It is relevant to notice that all V/P top ranked portfolios registered a lower beta (ranging from 0.86 to 0.95) compared to the other factors (ranging from 0.89 to 1.38). However, looking at IC coefficients (APPENDIX B, Table - B5), contrary to US it can be inferred that V/P is not the best return forecaster among all factors, posting a statistically significant IC coefficient only with 3 years forecasting horizon (ICs between 3% and 9%). On the other side both PEG and EV/EBITDA showed statistically significant ICs either with 6 months, 12 months and 36 months forecasting horizon (ICs between 5% and 14%). Nevertheless, we will not focus on all yearly returns' differences (details in APPENDIX B), rather we want to stress the superior returns obtained through a long-short strategy, which is self-financing. Graph 6 shows the cumulative long-short portfolios' performances of all factors (that use forward estimates) analyzed in the period 1995-2018.

Graph 6 – (1995-2018) Europe - Cumulative returns of monthly rebalanced portfolios: Long-short RIM-based V/P, long-short multiple's factors



Source: FactSet Research Systems (Alpha Testing), Bocconi University databases, our estimates

At first glance, specular to the US chart, it can be noticed a significant spread between long-short performances of the "V/P (Quintile)" multiple compared to the TP/P multiple, which represents analyst recommendations. As outlined by academic literature, analysts' biases seem to be at work also in Europe, considering that our model - based on analysts' estimates of EPS and DPS - consistently beat analysts' buy/sell recommendations. Furthermore, as in US, analysts' recommendations long-short spread returns did not show any significant correlation with other valuation models across the full period 1995-2018 (details APPENDIX B, Table - B7, F1-FN return correlations). Analysts did not exhibit differential target price forecasting ability as the last quintile portfolio (the least recommended stocks) outperformed the first quintile portfolio (the most recommended stocks) and through their target prices produced a negative yearly compounded long-short return of -2.8%. While TP/P multiple produces the worst strategy in our sample for the full period (1995-2018), it is important to notice that in the last sub-period (2010-2018) it added value producing a yearly spread long-short return of 4.2%, signaling differential target price forecasting ability. Furthermore, in Europe the strategy based on analysts' recommendations was the best performing in the last years (2010-2018) posting also the highest Sharpe ratio among all factors of 1.08 and a very low beta of 0.73.

Contrary to the US, in the last years not all strategies showed positive returns with classical "value signaling factors" (low P/E, PEG, P/B) performing the worst compared to "growth signaling factors" (high P/E, PEG, P/B). It is crucial to point out that the RIM-based V/P continued to show differential forecasting ability also in the period 2010-2018 (details in APPENDIX D), with the top quintile portfolio (high V/P) significantly outperforming the bottom quintile portfolio (low V/P). The results in our sample confirm our previous claim that "value" may not be signaled by low multiples (value trap) while a full valuation model could produce a better estimate. Therefore, RIM could be taken into consideration to study the "value anomaly" or "value factor". We leave open to further studies on stocks returns the possibility to add RIM in a multi-factor model (APT scenario) to the already known risk factors.

6. Conclusions

Our research highlighted the return forecasting ability of a residual income model based on analysts' estimates and time-varying risk-free rates, equity risk premiums and terminal growths, spanning from 1995 to 2018 in US and Europe. Three main results have been unveiled: a) RIM-based V/P portfolios outperformed main market indexes producing statistically significant alphas and low betas; b) they overcame portfolios built through other factors (main market multiples tied to "value premium") reporting higher Sharpe ratios and information ratios, with better evidences in US compared to Europe c) they remarkably beat analysts' buy-sell recommendations. In accordance with previous studies we confirmed the relevance of RIM as a sound valuation technique and stressed the paradox of analysts' forecasting returns inaccuracy as opposed to capability of producing reliable financial estimates. We displayed that the use of a residual income model-based valuation could remarkably improve the analysts' price target quality. That being said, analysts demonstrated significantly improved target price forecasting ability in the period 2010-2018, especially in Europe. The most surprising outcome has been the ability of RIM-based V/P portfolios to achieve substantial long-short returns, along the total time span and in all the sub-periods excluding few and small exceptions, leading in particular to the identification of the most overvalued stocks. Another noticeable result resides into the superior return forecasting ability shown by the simpler V/P model with a truncation at the third year of analysts' estimates compared with the one with additional growth of 5 years through sustainable growth. At the same time, they leave open the question about why, after all these evidences, the model still enjoys low consideration in the practitioners' community, especially among the markets' operators, which should contribute to market efficiency through their trades. In a framework where market inefficiencies are admitted, a valuation model that shows superior predicting power for returns, at least compared to main market multiples and analysts' recommendations, should be considered in providing better empirical estimates of intrinsic value.

Sources for all the tables: FactSet Research Systems (Alpha Testing), Bocconi University databases.

APPENDIX A – (1995-2018) United States - Full Statistics on monthly equally-weighted rebalanced portfolios

Table – A1 - US. Long-short yearly returns (F1-FN), long-only yearly returns for quintiles and top/bottom 20-30 stocks (only V/P), benchmark yearly return (S&P 500), Sharpe ratios for quintiles and top/bottom 20-30 stocks (only V/P)

		Universe Return						Sharpe Ratio				
Factor	F1-FN	1	2	3	4	5	Benchmark Return	1	2	3	4	5
F. V/P (Quintile)	7,65	13,05	12,18	12,38	7,63	2,89	8,66	0,82	0,85	0,78	0,41	0,14
F. V/P (Top 20)	12,61	15,18	10,09	-0,79			8,66	0,80	0,64	-0,03		
F. V/P (Top 30)	9,47	15,11	10,02	2,44			8,66	0,84	0,64	0,10		
F. V/P (+5Y) (Quintile)	5,43	12,62	11,93	12,06	7,44	4,50	8,66	0,84	0,82	0,78	0,43	0,21
F. V/P (+5Y) (Top 20)	6,86	13,02	10,07	3,06			8,66	0,75	0,64	0,13		
F. V/P (+5Y) (Top 30)	5,83	12,57	10,22	3,52			8,66	0,75	0,66	0,15		
F. Target Price / P	-3,48	8,49	10,12	11,33	10,74	10,75	8,66	0,46	0,60	0,68	0,63	0,57
F. ROE (Trailing)	-0,24	6,58	10,40	7,80	9,63	7,59	8,66	0,29	0,62	0,46	0,67	0,52
F. ROE (Forward)	6,71	13,56	11,80	10,23	8,49	4,56	8,66	0,59	0,76	0,66	0,56	0,20
F. P/E (Trailing)	-5,61	5,03	9,39	11,20	11,18	11,73	8,66	0,24	0,61	0,72	0,71	0,71
F. P/E (Forward)	0,12	8,43	11,29	11,11	8,67	8,57	8,66	0,30	0,72	0,76	0,58	0,43
F. PEG (Trailing)	-0,77	8,15	10,54	11,44	9,82	7,67	8,66	0,41	0,67	0,80	0,65	0,35
F. PEG (Forward)	-2,09	5,77	10,61	12,51	9,78	8,52	8,66	0,26	0,55	0,85	0,70	0,52
F. P/B (Trailing)	-0,17	6,42	12,67	11,40	9,76	6,77	8,66	0,29	0,69	0,74	0,67	0,41
F. P/B (Forward)	0,88	11,31	10,85	9,48	8,92	7,58	8,66	0,46	0,69	0,59	0,54	0,35
F. EV/EBITDA Historical	0,28	8,11	9,81	8,80	8,60	6,06	8,66	0,44	0,64	0,61	0,56	0,28
F. EV/EBITDA Forward	0,28	8,11	9,81	8,80	8,60	6,06	8,66	0,44	0,64	0,61	0,56	0,28
F. Mkt Value Size	3,37	10,07	12,61	8,81	10,64	6,70	8,66	0,57	0,50	0,54	0,47	0,46

Table – A2 - US. Information ratios, alphas, alphas' t-stat for quintiles and top/bottom 20-30 stocks (only V/P)

	Information Ratio					Alpha					T-Stat Alpha				
Factor	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	0,46	0,54	0,59	-0,08	-0,43	0,42	0,35	0,30	-0,14	-0,53	2,89	3,70	2,98	-1,15	-2,81
F. V/P (Top 20)	0,50	0,31	-0,52			0,57	0,11	-0,86			2,69	1,40	-3,40		
F. V/P (Top 30)	0,54	0,31	-0,36			0,56	0,10	-0,58			2,94	1,36	-2,49		
F. V/P (+5Y) (Quintile)	0,52	0,47	0,61	-0,14	-0,28	0,38	0,32	0,28	-0,13	-0,41	3,28	3,24	3,16	-1,21	-2,11
F. V/P (+5Y) (Top 20)	0,44	0,30	-0,31			0,38	0,11	-0,54			2,34	1,46	-2,30		
F. V/P (+5Y) (Top 30)	0,43	0,34	-0,29			0,34	0,13	-0,51			2,33	1,72	-2,25		
F. Target Price / P	-0,01	0,26	0,40	0,23	0,22	0,01	0,08	0,20	0,19	0,15	0,06	0,85	1,79	1,31	0,88
F. ROE (Trailing)	0,09	0,72	0,23	0,36	0,14	-0,14	0,21	0,22	0,32	0,13	-0,76	2,42	1,23	2,23	1,06
F. ROE (Forward)	0,31	0,59	0,26	-0,05	-0,24	0,34	0,25	0,15	0,04	-0,39	1,28	2,93	1,55	0,39	-1,75
F. P/E (Trailing)	-0,25	0,09	0,43	0,45	0,46	-0,40	0,10	0,22	0,20	0,24	-2,27	0,96	2,36	2,23	2,17
F. P/E (Forward)	0,03	0,29	0,38	-0,02	0,02	-0,11	0,27	0,25	0,03	-0,02	-0,34	2,02	2,63	0,48	-0,12
F. PEG (Trailing)	-0,00	0,22	0,45	0,22	-0,03	-0,12	0,21	0,28	0,12	-0,14	-0,73	1,62	3,26	1,45	-0,65
F. PEG (Forward)	-0,17	0,26	0,59	0,11	-0,04	-0,35	0,07	0,36	0,19	0,08	-1,92	0,52	3,68	1,85	0,52
F. P/B (Trailing)	-0,12	0,50	0,43	0,11	-0,20	-0,31	0,26	0,25	0,17	-0,03	-1,73	1,92	2,52	1,54	-0,20
F. P/B (Forward)	0,16	0,31	0,15	0,06	-0,05	0,19	0,21	0,06	0,01	-0,05	0,65	1,86	0,68	0,11	-0,19
F. EV/EBITDA Historical	0,24	0,52	0,47	0,51	0,07	0,16	0,34	0,26	0,21	-0,07	0,97	2,70	2,66	2,35	-0,38
F. EV/EBITDA Forward	0,24	0,52	0,47	0,51	0,07	0,16	0,34	0,26	0,21	-0,07	0,97	2,70	2,66	2,35	-0,38
F. Mkt Value Size	0,19	0,22	0,03	0,13	-0,47	0,09	0,28	0,02	0,15	-0,11	0,66	0,89	0,16	0,56	-1,59

Table – A3 - US. Betas, betas' t-stat, R squared for quintiles and top/bottom 20-30 stocks (only V/P)

	Beta					T-Stat Beta					R^2 for Beta				
Factor	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	0,91	0,89	1,00	1,15	1,21	27,01	40,83	42,39	39,32	27,74	0,73	0,86	0,87	0,85	0,74
F. V/P (Top 20)	0,98	1,02	1,35			19,76	57,46	22,83			0,59	0,92	0,65		
F. V/P (Top 30)	0,96	1,02	1,29			21,44	59,32	23,79			0,63	0,93	0,67		
F. V/P (+5Y) (Quintile)	0,91	0,90	0,98	1,09	1,24	33,98	38,49	47,69	42,61	27,43	0,81	0,84	0,89	0,87	0,73
F. V/P (+5Y) (Top 20)	0,98	1,01	1,31			26,38	56,45	23,98			0,72	0,92	0,68		
F. V/P (+5Y) (Top 30)	0,98	1,00	1,32			28,63	57,12	24,86			0,75	0,92	0,69		
F. Target Price / P	1,03	1,07	1,04	1,00	1,09	25,17	46,59	40,27	29,63	28,07	0,70	0,89	0,86	0,76	0,74
F. ROE (Trailing)	1,47	1,13	0,97	0,88	0,93	44,66	#N/D	24,91	30,27	44,09	0,90	1,01	0,74	0,81	0,90
F. ROE (Forward)	1,15	1,00	0,98	0,94	1,25	18,70	49,54	44,82	38,05	23,91	0,56	0,90	0,88	0,84	0,68
F. P/E (Trailing)	1,27	0,95	0,98	1,00	1,02	31,32	38,15	45,01	46,88	40,02	0,78	0,84	0,88	0,89	0,85
F. P/E (Forward)	1,37	0,93	0,92	0,96	1,11	18,01	29,52	41,49	56,26	25,27	0,54	0,76	0,86	0,92	0,70
F. PEG (Trailing)	1,19	0,94	0,90	0,97	1,22	32,64	31,49	44,42	52,39	24,41	0,79	0,78	0,88	0,91	0,68
F. PEG (Forward)	1,31	1,18	0,92	0,85	0,91	30,53	35,29	40,48	35,64	24,51	0,77	0,82	0,86	0,82	0,69
F. P/B (Trailing)	1,32	1,12	0,96	0,88	0,89	31,61	34,95	41,42	33,44	21,84	0,78	0,82	0,86	0,80	0,63
F. P/B (Forward)	1,17	0,97	1,02	1,04	1,09	16,89	37,42	46,43	43,22	18,61	0,51	0,84	0,89	0,87	0,56
F. EV/EBITDA Historical	1,10	0,94	0,93	1,00	1,34	30,01	35,91	48,96	61,45	33,02	0,79	0,84	0,91	0,94	0,82
F. EV/EBITDA Forward	1,10	0,94	0,93	1,00	1,34	30,01	35,91	48,96	61,45	33,02	0,79	0,84	0,91	0,94	0,82
F. Mkt Value Size	1,08	1,20	1,02	1,11	0,94	34,74	16,51	38,95	17,63	58,20	0,81	0,50	0,85	0,53	0,92

Table – A4 - US. Hit-Rate % (percentage of successful bet), monthly turnover, maximum drawdown over the full period of analysis for quintiles and top/bottom 20-30 stocks (only V/P)

	Hit-Rate % > Bench					Turnover %					Maximum Drawdown (-100)				
Factor	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	57,76	55,96	58,12	49,10	46,57	22,07	35,99	37,43	32,68	20,46	-57,70	-46,11	-49,76	-63,88	-83,60
F. V/P (Top 20)	59,93	56,32	42,60			36,30	6,56	27,14			-63,59	-52,20	-93,60		
F. V/P (Top 30)	**60,65**	**55,60**	**44,77**			**32,85**	**7,40**	**26,00**			**-58,62**	**-52,17**	**-88,42**		
F. V/P (+5Y) (Quintile)	53,79	53,07	56,68	49,10	47,29	17,36	29,61	32,04	27,67	16,33	-48,16	-51,23	-52,34	-53,56	-80,66
F. V/P (+5Y) (Top 20)	**56,68**	**53,07**	**49,82**			**20,02**	**5,67**	**21,68**			**-42,71**	**-53,04**	**-88,10**		
F. V/P (+5Y) (Top 30)	52,35	55,23	46,57			20,40	6,29	21,26			-49,16	-52,58	-87,06		
F. Target Price / P	**57,40**	**53,43**	**57,04**	**53,43**	**56,32**	**11,49**	**19,44**	**22,18**	**19,96**	**12,11**	**-68,00**	**-48,73**	**-49,39**	**-53,24**	**-70,44**
F. ROE (Trailing)	52,58	58,88	53,42	57,21	54,46	33,23	58,75	60,97	55,82	29,83	-68,09	-50,99	-46,91	-49,67	-54,23
F. ROE (Forward)	**53,79**	**60,29**	**54,87**	**50,90**	**48,74**	**4,04**	**5,71**	**6,93**	**7,49**	**7,30**	**-47,94**	**-48,66**	**-52,95**	**-58,43**	**-81,68**
F. P/E (Trailing)	47,29	47,65	56,32	55,23	58,12	9,16	13,17	14,46	13,15	7,70	-77,39	-53,70	-48,90	-48,01	-51,02
F. P/E (Forward)	**46,21**	**55,60**	**56,32**	**50,90**	**54,15**	**12,28**	**18,00**	**22,13**	**19,33**	**11,58**	**-76,40**	**-55,49**	**-49,53**	**-49,02**	**-66,46**
F. PEG (Trailing)	53,43	54,87	58,84	52,35	50,18	13,88	23,88	26,89	23,02	13,00	-68,38	-53,27	-44,05	-48,11	-74,46
F. PEG (Forward)	**46,21**	**51,62**	**59,57**	**56,68**	**53,07**	**18,87**	**26,95**	**34,01**	**32,36**	**23,44**	**-71,29**	**-60,36**	**-47,26**	**-47,44**	**-57,16**
F. P/B (Trailing)	49,10	57,76	60,29	54,15	47,29	20,23	30,64	34,96	32,49	23,29	-69,80	-55,03	-51,14	-44,36	-64,22
F. P/B (Forward)	**53,79**	**57,76**	**54,15**	**49,46**	**53,07**	**9,85**	**16,85**	**18,69**	**16,18**	**9,27**	**-70,42**	**-51,65**	**-49,50**	**-50,88**	**-77,41**
F. EV/EBITDA Historical	54,36	57,26	55,60	53,53	53,53	12,50	18,52	20,01	18,01	10,88	-57,62	-48,05	-51,51	-46,81	-71,87
F. EV/EBITDA Forward	**54,36**	**57,26**	**55,60**	**53,53**	**53,53**	**12,50**	**18,52**	**20,01**	**18,01**	**10,88**	**-57,62**	**-48,05**	**-51,51**	**-46,81**	**-71,87**
F. Mkt Value Size	54,15	48,38	49,46	48,01	41,88	23,89	19,74	15,03	9,68	3,51	-55,39	-58,98	-54,45	-48,67	-56,61

Table – A5 - US. Information coefficients (ICs) for quintiles and top/bottom 20-30 stocks (only V/P)

	Pooled Information Coefficient																			
	1				2				3				4				5			
Factor	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M
F. V/P (Quintile)	**0,06**	**0,12**	**0,15**	**0,15**	**0,04**	**0,09**	**0,13**	**0,09**	**0,03**	**0,09**	**0,13**	**0,14**	**0,04**	**0,11**	**0,17**	**0,23**	**0,03**	**0,06**	**0,09**	**0,16**
F. V/P (Top 20)	0,05	0,14	0,16	0,14	0,03	0,07	0,09	0,14	0,04	0,05	0,07	0,22								
F. V/P (Top 30)	**0,05**	**0,13**	**0,15**	**0,15**	**0,03**	**0,07**	**0,10**	**0,15**	**0,04**	**0,06**	**0,10**	**0,23**								
F. V/P (+5Y) (Quintile)	0,01	0,03	0,04	0,04	0,04	0,08	0,10	0,07	0,03	0,08	0,11	0,11	0,03	0,10	0,13	0,19	0,02	0,04	0,06	0,10
F. V/P (+5Y) (Top 20)	**-0,03**	**-0,05**	**-0,06**	**-0,06**	**0,02**	**0,06**	**0,08**	**0,11**	**-0,01**	**-0,04**	**-0,06**	**-0,14**								
F. V/P (+5Y) (Top 30)	-0,02	-0,04	-0,05	-0,03	0,02	0,06	0,08	0,11	0,00	-0,02	-0,02	-0,03								
F. Target Price / P	**-0,03**	**-0,07**	**-0,10**	**-0,12**	**-0,07**	**-0,17**	**-0,25**	**-0,33**	**-0,07**	**-0,18**	**-0,27**	**-0,35**	**-0,05**	**-0,14**	**-0,20**	**-0,30**	**-0,03**	**-0,10**	**-0,14**	**-0,22**
F. ROE (Trailing)	-0,01	-0,01	0,00	0,05	-0,00	-0,01	0,00	0,12	0,00	-0,03	-0,01	0,11	0,01	-0,02	0,00	0,11	0,02	0,01	0,02	0,11
F. ROE (Forward)	**-0,01**	**-0,03**	**-0,03**	**-0,00**	**-0,02**	**-0,04**	**-0,05**	**-0,01**	**-0,02**	**-0,05**	**-0,06**	**-0,01**	**-0,03**	**-0,07**	**-0,09**	**-0,10**	**0,01**	**0,03**	**0,04**	**0,08**
F. P/E (Trailing)	-0,01	-0,01	-0,04	-0,04	0,03	0,07	0,10	0,19	0,04	0,09	0,11	0,18	0,03	0,08	0,09	0,15	0,01	0,04	0,06	0,05
F. P/E (Forward)	**-0,03**	**-0,05**	**-0,06**	**-0,05**	**0,05**	**0,12**	**0,18**	**0,24**	**0,05**	**0,12**	**0,15**	**0,25**	**0,05**	**0,12**	**0,15**	**0,26**	**0,02**	**0,05**	**0,10**	**0,20**
F. PEG (Trailing)	0,00	0,02	0,02	0,03	0,06	0,14	0,22	0,28	0,06	0,13	0,20	0,30	0,05	0,11	0,19	0,30	0,03	0,07	0,12	0,20
F. PEG (Forward)	**0,01**	**0,05**	**0,05**	**0,08**	**0,03**	**0,07**	**0,07**	**0,15**	**0,03**	**0,04**	**0,04**	**0,17**	**0,02**	**0,04**	**0,05**	**0,16**	**0,01**	**0,01**	**0,04**	**0,10**
F. P/B (Trailing)	0,01	0,06	0,08	0,10	0,03	0,07	0,10	0,22	0,03	0,05	0,08	0,24	0,03	0,04	0,07	0,22	0,03	0,06	0,08	0,15
F. P/B (Forward)	**0,02**	**0,06**	**0,10**	**0,16**	**0,04**	**0,11**	**0,16**	**0,31**	**0,06**	**0,16**	**0,24**	**0,35**	**0,05**	**0,13**	**0,19**	**0,29**	**0,03**	**0,08**	**0,14**	**0,23**
F. EV/EBITDA Historical	0,03	0,06	0,04	0,01	0,06	0,12	0,15	0,20	0,05	0,13	0,16	0,23	0,06	0,12	0,15	0,28	0,06	0,13	0,16	0,21
F. EV/EBITDA Forward	**0,03**	**0,06**	**0,04**	**0,01**	**0,06**	**0,12**	**0,15**	**0,20**	**0,05**	**0,13**	**0,16**	**0,23**	**0,06**	**0,12**	**0,15**	**0,28**	**0,06**	**0,13**	**0,16**	**0,21**
F. Mkt Value Size	0,02	0,06	0,08	0,09	0,03	0,09	0,11	0,13	0,04	0,10	0,13	0,15	0,03	0,07	0,10	0,10	0,03	0,07	0,08	0,09

Table – A6 - US. Information coefficients' t-stat for quintiles and top/bottom 20-30 stocks (only V/P)

	Pooled IC T-Stat																			
	1				2				3				4				5			
Factor	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M
F. V/P (Quintile)	**9,40**	**18,38**	**22,67**	**21,05**	**6,85**	**14,38**	**19,85**	**12,57**	**5,48**	**14,16**	**19,86**	**20,45**	**6,21**	**18,03**	**25,90**	**32,81**	**4,89**	**10,06**	**13,18**	**21,96**
F. V/P (Top 20)	3,90	10,60	11,43	9,53	9,75	22,37	30,64	43,67	3,05	3,74	5,28	14,60								
F. V/P (Top 30)	**4,44**	**12,16**	**13,16**	**12,39**	**9,47**	**22,59**	**30,86**	**43,03**	**3,66**	**5,56**	**9,13**	**18,59**								
F. V/P (+5Y) (Quintile)	1,92	4,23	5,69	5,49	6,22	13,19	14,93	9,20	5,04	12,27	17,59	15,66	5,09	15,10	20,00	27,57	3,01	5,73	8,90	13,33
F. V/P (+5Y) (Top 20)	**-2,00**	**-3,79**	**-4,62**	**-4,13**	**8,41**	**19,52**	**26,06**	**33,07**	**-0,67**	**-2,93**	**-4,46**	**-9,33**								
F. V/P (+5Y) (Top 30)	-1,98	-3,66	-4,12	-2,79	8,22	18,86	25,45	32,82	0,06	-2,00	-2,01	-2,27								
F. Target Price / P	**-4,09**	**-11,01**	**-15,25**	**-17,96**	**-11,13**	**-26,69**	**-39,79**	**-49,39**	**-11,59**	**-29,07**	**-42,93**	**-52,29**	**-8,18**	**-22,65**	**-32,06**	**-43,85**	**-5,35**	**-16,25**	**-22,56**	**-32,69**
F. ROE (Trailing)	-1,33	-2,01	0,17	6,87	-0,57	-0,92	0,58	17,30	0,38	-4,95	-1,54	15,26	1,73	-3,49	0,29	14,96	3,11	0,89	2,47	15,35
F. ROE (Forward)	**-1,98**	**-4,46**	**-5,57**	**-0,23**	**-2,91**	**-6,34**	**-9,01**	**-2,29**	**-4,29**	**-8,22**	**-10,79**	**-1,05**	**-5,72**	**-12,12**	**-16,45**	**-16,70**	**2,13**	**4,72**	**6,56**	**12,81**
F. P/E (Trailing)	-1,57	-2,55	-6,04	-5,56	4,42	12,46	16,49	29,27	6,21	15,59	18,56	29,14	4,70	14,13	15,15	23,24	2,38	7,62	10,66	7,24
F. P/E (Forward)	**-4,76**	**-8,46**	**-10,79**	**-7,63**	**8,37**	**20,95**	**32,13**	**40,08**	**9,85**	**20,99**	**26,67**	**41,38**	**9,61**	**21,02**	**26,93**	**43,98**	**3,20**	**8,70**	**17,79**	**31,52**
F. PEG (Trailing)	0,08	3,14	3,77	4,21	10,65	23,82	37,68	44,71	10,71	23,28	35,19	50,13	8,13	18,96	32,07	49,60	5,19	12,66	20,61	32,63
F. PEG (Forward)	**2,19**	**7,83**	**7,47**	**10,22**	**4,30**	**10,82**	**11,05**	**20,90**	**4,16**	**5,93**	**6,23**	**23,62**	**2,58**	**5,83**	**6,85**	**22,53**	**0,92**	**1,97**	**5,45**	**12,97**
F. P/B (Trailing)	0,78	9,78	12,38	13,10	5,15	11,41	14,60	31,02	4,70	7,99	12,08	34,14	4,40	6,50	11,17	31,13	3,96	9,44	11,44	20,18
F. P/B (Forward)	**3,69**	**10,90**	**17,54**	**25,66**	**7,99**	**18,81**	**28,41**	**51,56**	**10,47**	**28,37**	**41,85**	**59,93**	**9,48**	**23,42**	**34,05**	**48,38**	**4,63**	**14,01**	**23,58**	**37,03**
F. EV/EBITDA Historical	4,34	8,48	5,99	1,39	8,65	18,16	22,84	27,42	8,29	19,07	23,90	31,45	8,44	18,47	22,01	39,59	8,76	19,25	23,73	29,20
F. EV/EBITDA Forward	**4,34**	**8,48**	**5,99**	**1,39**	**8,65**	**18,16**	**22,84**	**27,42**	**8,29**	**19,07**	**23,90**	**31,45**	**8,44**	**18,47**	**22,01**	**39,59**	**8,76**	**19,25**	**23,73**	**29,20**
F. Mkt Value Size	4,43	11,36	14,93	14,63	6,20	16,24	19,06	21,20	6,74	17,95	22,53	24,01	5,70	13,24	17,14	16,06	5,54	12,27	15,08	14,90

Table – A7 - US. Long-short (F1-FN) spread return correlation for quintiles and top/bottom 20-30 stocks (only V/P)

	F1-FN Return Correlation																	
Factor	V/P (Quintile)	V/P (Top 20)	V/P (Top 30)	V/P (+5Y) (Quintile)	V/P (+5Y) (Top 20)	V/P (+5Y) (Top 30)	Target Price / P	ROE (Trailing)	ROE (Forward)	P/E (Trailing)	P/E (Forward)	PEG (Trailing)	PEG (Forward)	P/B (Trailing)	P/B (Forward)	EV/EBITDA Historical	EV/EBITDA Forward	Mkt Value Size
F. V/P (Quintile)	1,00																	
F. V/P (Top 20)	0,95	1,00																
F. V/P (Top 30)	0,94	0,94	1,00															
F. V/P (+5Y) (Quintile)	0,93	0,88	0,88	1,00														
F. V/P (+5Y) (Top 20)	0,47	0,46	0,42	0,66	1,00													
F. V/P (+5Y) (Top 30)	0,72	0,69	0,72	0,87	0,86	1,00												
F. Target Price / P	-0,20	-0,18	-0,19	-0,23	-0,20	-0,23	1,00											
F. ROE (Trailing)	-0,16	-0,09	-0,04	-0,25	-0,17	-0,23	-0,30	1,00										
F. ROE (Forward)	0,01	0,00	0,03	-0,01	-0,05	-0,01	-0,05	0,16	1,00									
F. P/E (Trailing)	-0,29	-0,30	-0,26	-0,37	-0,34	-0,36	-0,02	0,39	0,45	1,00								
F. P/E (Forward)	0,01	0,04	0,04	0,01	0,03	0,00	-0,03	0,12	0,00	0,01	1,00							
F. PEG (Trailing)	0,37	0,35	0,30	0,45	0,28	0,34	-0,37	0,28	0,04	-0,05	0,08	1,00						
F. PEG (Forward)	-0,01	-0,04	0,01	-0,09	-0,12	-0,10	-0,19	0,55	0,31	0,63	0,04	0,18	1,00					
F. P/B (Trailing)	0,22	0,17	0,23	0,15	-0,01	0,09	-0,25	0,63	0,33	0,34	0,10	0,39	0,71	1,00				
F. P/B (Forward)	0,05	0,08	0,08	0,05	0,04	0,03	-0,04	0,12	0,00	-0,00	1,00	0,09	0,04	0,11	1,00			
F. EV/EBITDA Historical	0,28	0,32	0,25	0,36	0,25	0,28	-0,17	-0,20	-0,05	-0,19	0,04	0,66	0,11	0,24	0,05	1,00		
F. EV/EBITDA Forward	0,28	0,32	0,25	0,36	0,25	0,28	-0,17	-0,20	-0,05	-0,19	0,04	0,66	0,11	0,24	0,05	1,00	1,00	
F. Mkt Value Size	-0,12	-0,10	-0,11	-0,22	-0,15	-0,17	0,08	0,25	0,16	0,33	-0,06	-0,05	0,28	0,26	-0,07	-0,00	-0,00	1,00

APPENDIX B – (1995-2018) Europe - Full Statistics on monthly equally-weighted rebalanced portfolios

Table – B1 - EU. Long-short yearly returns (F1-FN), long-only yearly returns for quintiles and top/bottom 20-30 stocks (only V/P), benchmark yearly return (S&P 500), Sharpe ratios for quintiles and top/bottom 20-30 stocks (only V/P)

		Universe Return						Sharpe Ratio				
Factor	F1-FN	1	2	3	4	5	Benchmark Return	1	2	3	4	5
F. V/P (Quintile)	4,55	11,54	9,95	8,93	7,66	5,18	4,11	0,71	0,66	0,58	0,45	0,27
F. V/P (Top 20)	4,59	10,49	8,89	3,60			4,11	0,55	0,57	0,17		
F. V/P (Top 30)	4,14	10,64	8,91	4,21			4,11	0,59	0,57	0,20		
F. V/P (+5Y) (Quintile)	3,23	10,85	9,87	9,03	7,84	6,09	4,11	0,71	0,66	0,59	0,47	0,32
F. V/P (+5Y) (Top 20)	2,23	9,10	8,99	4,35			4,11	0,55	0,57	0,20		
F. V/P (+5Y) (Top 30)	3,06	9,38	9,09	4,33			4,11	0,57	0,58	0,21		
F. Target Price / P	-2,81	8,66	8,88	8,46	8,24	9,70	4,11	0,50	0,61	0,55	0,50	0,47
F. ROE (Trailing)	0,19	3,64	6,63	7,81	6,82	3,99	4,11	0,16	0,39	0,52	0,48	0,26
F. ROE (Forward)	1,38	9,43	9,82	9,27	9,30	7,29	4,11	0,64	0,64	0,61	0,58	0,42
F. P/E (Trailing)	-3,71	6,05	8,10	9,41	9,99	10,51	4,11	0,31	0,49	0,60	0,68	0,71
F. P/E (Forward)	0,62	8,41	10,02	9,24	9,18	7,80	4,11	0,42	0,63	0,65	0,66	0,47
F. PEG (Trailing)	3,79	10,20	8,88	9,51	9,26	5,89	4,11	0,50	0,57	0,66	0,65	0,32
F. PEG (Forward)	0,01	7,40	9,59	9,14	9,75	7,81	4,11	0,38	0,50	0,58	0,72	0,54
F. P/B (Trailing)	0,89	7,92	9,94	9,09	9,20	7,54	4,11	0,38	0,53	0,60	0,67	0,54
F. P/B (Forward)	0,59	9,67	8,43	8,58	9,27	8,49	4,11	0,48	0,52	0,58	0,66	0,51
F. EV/EBITDA Historical	4,48	11,35	10,16	10,79	8,83	5,99	4,11	0,69	0,68	0,78	0,62	0,35
F. EV/EBITDA Forward	2,69	9,43	8,82	7,67	5,45	5,72	4,11	0,55	0,59	0,52	0,37	0,31
F. Size	1,62	9,16	9,93	9,74	9,01	7,16	4,11	0,59	0,64	0,63	0,57	0,45

Table – B2 - EU. Information ratios, alphas, Alphas' t-stat for quintiles and top/bottom 20-30 stocks (only V/P)

	Information Ratio					Alpha					T-Stat Alpha				
Factor	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	0,82	0,89	0,82	0,65	0,15	0,63	0,50	0,41	0,29	0,08	4,38	4,82	4,39	3,03	0,57
F. V/P (Top 20)	0,51	1,06	-0,01			0,57	0,39	-0,00			2,75	5,22	-0,02		
F. V/P (Top 30)	0,55	1,06	0,04			0,58	0,39	0,03			3,03	5,25	0,17		
F. V/P (+5Y) (Quintile)	0,86	0,86	0,92	0,73	0,28	0,58	0,49	0,41	0,30	0,15	4,71	4,78	4,79	3,39	1,12
F. V/P (+5Y) (Top 20)	0,45	1,06	0,06			0,47	0,40	0,04			2,77	5,25	0,18		
F. V/P (+5Y) (Top 30)	0,56	1,08	0,07			0,47	0,41	0,02			3,15	5,38	0,11		
F. Target Price / P	0,50	0,77	0,80	0,61	0,56	0,40	0,42	0,37	0,35	0,44	2,63	4,52	4,29	3,09	2,53
F. ROE (Trailing)	0,38	1,27	1,19	0,90	0,46	0,40	0,56	0,66	0,57	0,34	1,89	5,41	5,24	4,21	2,21
F. ROE (Forward)	0,86	1,02	0,93	0,94	0,51	0,46	0,47	0,44	0,42	0,26	4,85	5,29	4,99	4,62	2,38
F. P/E (Trailing)	0,27	0,72	0,97	1,02	0,96	0,14	0,32	0,44	0,50	0,54	1,04	3,44	4,94	5,61	5,32
F. P/E (Forward)	0,56	0,81	0,79	0,91	0,48	0,33	0,50	0,45	0,44	0,32	2,40	4,25	4,92	5,72	2,51
F. PEG (Trailing)	0,69	0,65	0,86	1,00	0,22	0,46	0,41	0,47	0,44	0,16	3,06	3,59	5,10	5,88	1,09
F. PEG (Forward)	0,46	0,72	0,91	0,91	0,55	0,25	0,42	0,42	0,50	0,34	1,96	3,24	4,60	5,97	3,53
F. P/B (Trailing)	0,46	0,83	0,95	0,82	0,48	0,29	0,45	0,42	0,45	0,32	1,93	3,75	5,03	5,36	3,40
F. P/B (Forward)	0,62	0,66	0,77	0,83	0,44	0,43	0,37	0,39	0,46	0,40	2,77	3,42	4,36	5,22	2,56
F. EV/EBITDA Historical	1,08	0,86	0,99	0,73	0,24	0,58	0,52	0,58	0,42	0,17	5,24	4,80	6,15	4,51	1,30
F. EV/EBITDA Forward	1,00	1,03	1,21	0,60	0,53	0,63	0,58	0,55	0,32	0,34	4,48	4,99	5,94	3,06	2,39
F. Size	0,77	0,87	1,02	1,05	0,86	0,43	0,49	0,47	0,40	0,24	4,08	4,61	5,21	5,11	4,05

Table – B3 - EU. Betas, betas' t-stat, R squared for quintiles and top/bottom 20-30 stocks (only V/P)

	Beta					T-Stat Beta					R^2 for Beta				
Factor	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	0,90	0,91	0,93	1,03	1,14	27,66	39,16	44,46	48,97	34,06	0,74	0,85	0,88	0,90	0,81
F. V/P (Top 20)	0,95	0,98	1,14			20,35	58,07	23,29			0,60	0,92	0,66		
F. V/P (Top 30)	0,91	0,98	1,13			21,11	58,31	25,31			0,62	0,93	0,70		
F. V/P (+5Y) (Quintile)	0,89	0,89	0,94	1,04	1,14	32,26	38,37	48,89	53,16	37,54	0,79	0,84	0,90	0,91	0,84
F. V/P (+5Y) (Top 20)	0,86	0,98	1,20			22,53	57,37	25,68			0,65	0,92	0,71		
F. V/P (+5Y) (Top 30)	0,90	0,97	1,20			27,02	57,03	30,47			0,73	0,92	0,77		
F. Target Price / P	0,97	0,89	0,94	0,99	1,16	28,59	42,90	48,41	38,91	29,52	0,75	0,87	0,89	0,85	0,76
F. ROE (Trailing)	1,38	1,09	0,91	0,83	0,87	32,32	68,16	38,31	30,80	27,12	0,83	0,96	0,87	0,81	0,77
F. ROE (Forward)	0,89	0,94	0,92	0,98	1,05	42,36	46,48	47,08	47,60	43,04	0,87	0,89	0,89	0,89	0,87
F. P/E (Trailing)	1,17	1,01	0,95	0,90	0,89	37,85	47,83	47,91	45,23	38,56	0,84	0,89	0,89	0,88	0,84
F. P/E (Forward)	1,18	0,93	0,85	0,85	0,98	38,65	35,22	41,02	49,09	34,17	0,84	0,82	0,86	0,90	0,81
F. PEG (Trailing)	1,21	0,92	0,87	0,88	1,05	35,37	35,40	42,04	52,46	32,42	0,82	0,82	0,87	0,91	0,79
F. PEG (Forward)	1,15	1,14	0,96	0,83	0,87	40,28	39,08	47,10	44,28	40,49	0,86	0,85	0,89	0,88	0,86
F. P/B (Trailing)	1,22	1,13	0,93	0,83	0,84	36,87	42,04	50,17	44,00	39,02	0,83	0,87	0,90	0,88	0,85
F. P/B (Forward)	1,16	0,97	0,90	0,85	0,91	32,88	40,34	45,35	43,19	26,05	0,80	0,86	0,88	0,87	0,71
F. EV/EBITDA Historical	0,99	0,89	0,82	0,86	1,01	39,32	36,55	39,07	41,20	33,49	0,85	0,83	0,85	0,86	0,80
F. EV/EBITDA Forward	1,01	0,90	0,89	0,91	1,11	35,99	39,01	42,69	47,52	40,10	0,84	0,86	0,88	0,90	0,87
F. Size	0,94	0,92	0,94	0,99	1,01	39,78	38,36	47,01	56,71	74,60	0,85	0,84	0,89	0,92	0,95

Table – B4 - EU. Hit-Rate % (percentage of successful bet), monthly turnover, maximum drawdown over the full period of analysis for quintiles and top/bottom 20-30 stocks (only V/P)

	Hit-Rate % > Bench					Turnover %					Maximum Drawdown (-100)				
Factor	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	63,18	65,70	62,45	61,01	53,79	22,14	39,06	41,28	34,04	18,11	-62,59	-53,25	-50,86	-58,93	-76,27
F. V/P (Top 20)	56,68	64,62	55,23			37,99	4,57	24,93			-72,31	-53,42	-80,74		
F. V/P (Top 30)	59,93	65,34	55,23			34,76	5,43	22,69			-70,13	-53,04	-80,93		
F. V/P (+5Y) (Quintile)	63,54	66,06	63,54	60,65	57,40	17,41	31,99	35,26	27,46	14,40	-60,24	-53,04	-53,00	-57,96	-70,46
F. V/P (+5Y) (Top 20)	57,04	66,79	54,87			22,95	3,81	20,62			-54,88	-54,45	-73,64		
F. V/P (+5Y) (Top 30)	57,40	65,70	53,43			23,12	4,46	18,83			-57,10	-53,94	-76,82		
F. Target Price / P	61,01	61,37	62,45	59,93	62,45	8,40	15,44	18,31	17,69	9,90	-71,96	-49,07	-53,61	-59,65	-68,98
F. ROE (Trailing)	60,83	70,05	68,75	67,27	59,91	32,91	57,23	59,02	51,55	25,07	-66,39	-58,02	-51,27	-46,78	-56,56
F. ROE (Forward)	65,70	63,18	66,43	64,98	54,51	4,25	5,92	6,38	6,45	5,42	-54,54	-54,73	-55,60	-56,23	-55,72
F. P/E (Trailing)	53,43	58,84	64,26	66,06	68,59	8,31	13,07	13,54	11,90	7,02	-62,82	-58,70	-53,65	-52,40	-47,26
F. P/E (Forward)	56,68	64,98	65,70	61,73	61,73	10,65	17,56	21,26	18,75	11,58	-65,76	-56,17	-49,70	-47,43	-67,91
F. PEG (Trailing)	60,65	63,90	63,18	62,82	54,87	14,22	26,06	28,24	24,05	13,67	-68,87	-56,95	-48,09	-50,19	-74,56
F. PEG (Forward)	56,32	63,90	63,54	62,82	59,93	17,78	26,38	34,48	33,49	24,81	-57,41	-64,11	-58,25	-45,44	-58,46
F. P/B (Trailing)	58,12	61,01	61,73	63,90	58,12	20,05	28,82	34,77	33,24	23,72	-60,93	-62,30	-53,33	-49,53	-51,93
F. P/B (Forward)	60,65	61,73	61,73	62,82	60,65	8,95	15,78	16,44	14,16	8,04	-67,01	-58,93	-51,59	-44,26	-71,14
F. EV/EBITDA Historical	66,06	64,98	66,79	62,09	55,96	6,51	8,88	9,57	8,73	6,34	-50,58	-49,39	-42,38	-49,11	-74,41
F. EV/EBITDA Forward	66,67	63,37	65,59	57,20	60,08	9,40	15,51	16,26	15,01	9,13	-54,27	-48,19	-46,21	-56,82	-71,65
F. Size	60,29	64,26	64,62	66,43	65,34	19,32	15,53	11,86	7,96	3,06	-56,14	-51,14	-52,72	-54,91	-52,33

Table – B5 - EU. Information coefficients (ICs) for quintiles and top/bottom 20-30 stocks (only V/P)

	Pooled Information Coefficient																			
	1				2				3				4				5			
Factor	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M
F. V/P (Quintile)	**-0,02**	**-0,06**	**-0,03**	**0,06**	**-0,05**	**-0,07**	**-0,04**	**0,13**	**-0,04**	**-0,05**	**-0,02**	**0,17**	**-0,03**	**-0,03**	**-0,01**	**0,16**	**0,00**	**0,01**	**0,02**	**0,09**
F. V/P (Top 20)	0,00	-0,02	0,02	0,08	-0,00	-0,00	0,02	0,11	0,02	0,01	-0,01	0,02								
F. V/P (Top 30)	**-0,01**	**-0,02**	**0,01**	**0,08**	**-0,00**	**-0,00**	**0,02**	**0,12**	**0,03**	**0,02**	**0,00**	**0,04**								
F. V/P (+5Y) (Quintile)	-0,03	-0,05	-0,05	0,03	-0,04	-0,07	-0,07	0,09	-0,05	-0,07	-0,04	0,12	-0,04	-0,06	-0,04	0,13	-0,00	0,01	0,01	0,09
F. V/P (+5Y) (Top 20)	**-0,02**	**-0,02**	**-0,01**	**0,09**	**-0,00**	**-0,01**	**0,00**	**0,07**	**0,00**	**-0,03**	**-0,04**	**-0,05**								
F. V/P (+5Y) (Top 30)	-0,03	-0,05	-0,04	0,08	-0,01	-0,01	-0,00	0,07	0,01	0,01	-0,02	-0,00								
F. Target Price / P	**-0,02**	**-0,09**	**-0,14**	**-0,26**	**-0,04**	**-0,13**	**-0,20**	**-0,36**	**-0,04**	**-0,12**	**-0,20**	**-0,35**	**-0,01**	**-0,07**	**-0,14**	**-0,33**	**-0,00**	**-0,05**	**-0,09**	**-0,18**
F. ROE (Trailing)	-0,03	-0,06	-0,06	0,04	-0,05	-0,08	-0,06	0,10	-0,07	-0,09	-0,06	0,11	-0,04	-0,11	-0,06	0,11	-0,02	-0,04	-0,02	0,06
F. ROE (Forward)	**-0,01**	**-0,03**	**-0,04**	**-0,06**	**-0,05**	**-0,12**	**-0,17**	**-0,16**	**-0,06**	**-0,15**	**-0,20**	**-0,17**	**-0,06**	**-0,17**	**-0,22**	**-0,20**	**-0,01**	**-0,02**	**-0,01**	**0,06**
F. P/E (Trailing)	-0,00	0,01	0,01	0,00	0,01	0,06	0,10	0,24	0,02	0,07	0,10	0,22	0,02	0,07	0,09	0,22	0,01	0,03	0,04	0,09
F. P/E (Forward)	**-0,03**	**-0,04**	**-0,05**	**0,03**	**0,01**	**0,04**	**0,07**	**0,16**	**0,03**	**0,08**	**0,12**	**0,19**	**0,02**	**0,08**	**0,13**	**0,21**	**0,02**	**0,07**	**0,11**	**0,17**
F. PEG (Trailing)	-0,02	0,00	0,01	-0,01	0,02	0,07	0,12	0,23	0,03	0,10	0,16	0,28	0,03	0,10	0,17	0,31	0,03	0,08	0,13	0,22
F. PEG (Forward)	**0,02**	**0,06**	**0,06**	**0,12**	**-0,01**	**0,01**	**0,02**	**0,10**	**-0,00**	**0,01**	**0,01**	**0,12**	**0,00**	**0,03**	**0,03**	**0,16**	**-0,01**	**-0,01**	**0,01**	**0,07**
F. P/B (Trailing)	0,01	0,05	0,06	0,09	0,00	0,04	0,04	0,17	0,00	0,03	0,05	0,20	0,01	0,06	0,08	0,25	-0,00	0,01	0,02	0,13
F. P/B (Forward)	**0,00**	**0,03**	**0,07**	**0,14**	**0,01**	**0,08**	**0,15**	**0,34**	**0,04**	**0,13**	**0,20**	**0,35**	**0,04**	**0,14**	**0,21**	**0,37**	**0,02**	**0,09**	**0,13**	**0,24**
F. EV/EBITDA Historical	0,03	0,05	0,05	0,03	0,09	0,19	0,22	0,21	0,10	0,23	0,30	0,31	0,09	0,21	0,26	0,31	0,07	0,15	0,17	0,24
F. EV/EBITDA Forward	**0,04**	**0,09**	**0,12**	**0,09**	**0,08**	**0,18**	**0,20**	**0,17**	**0,09**	**0,20**	**0,24**	**0,28**	**0,10**	**0,23**	**0,28**	**0,31**	**0,04**	**0,10**	**0,12**	**0,16**
F. Size	0,04	0,11	0,15	0,17	0,04	0,11	0,14	0,17	0,05	0,14	0,17	0,19	0,05	0,14	0,19	0,21	0,06	0,14	0,19	0,23

Table – B6 - EU. Information coefficients' t-stat for quintiles and top/bottom 20-30 stocks (only V/P)

	Pooled IC T-Stat																			
	1				2				3				4				5			
Factor	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M	1 M	6 M	12 M	36 M
F. V/P (Quintile)	**-3,78**	**-9,13**	**-4,52**	**8,65**	**-7,58**	**-11,46**	**-6,69**	**19,12**	**-6,79**	**-8,78**	**-3,91**	**26,29**	**-4,98**	**-5,34**	**-1,05**	**24,26**	**0,47**	**2,43**	**2,93**	**13,39**
F. V/P (Top 20)	0,03	-1,19	1,46	4,99	-0,36	-0,23	5,99	35,33	1,58	0,59	-0,84	1,15								
F. V/P (Top 30)	**-0,84**	**-1,58**	**1,18**	**6,49**	**-1,19**	**-0,96**	**5,48**	**36,28**	**2,30**	**1,36**	**0,05**	**2,89**								
F. V/P (+5Y) (Quintile)	-4,90	-8,76	-7,28	3,81	-7,18	-12,21	-10,46	13,00	-7,84	-11,74	-7,13	18,56	-6,11	-9,28	-5,73	19,65	-0,16	1,35	2,33	13,56
F. V/P (+5Y) (Top 20)	**-1,16**	**-1,62**	**-0,86**	**6,07**	**-1,48**	**-3,29**	**0,81**	**21,05**	**0,03**	**-2,12**	**-3,18**	**-3,30**								
F. V/P (+5Y) (Top 30)	-2,38	-4,72	-3,36	6,25	-2,26	-4,41	-0,52	20,92	1,04	0,57	-1,54	-0,10								
F. Target Price / P	**-4,02**	**-15,86**	**-23,74**	**-42,58**	**-6,80**	**-22,70**	**-34,70**	**-60,46**	**-6,23**	**-20,39**	**-34,26**	**-57,56**	**-1,23**	**-12,69**	**-24,28**	**-53,00**	**-0,66**	**-7,63**	**-15,30**	**-27,98**
F. ROE (Trailing)	-5,13	-9,71	-8,82	5,73	-8,02	-12,66	-9,09	14,18	-9,98	-14,01	-8,40	15,50	-6,36	-16,30	-8,95	14,05	-2,62	-5,31	-3,03	7,61
F. ROE (Forward)	**-2,29**	**-5,99**	**-7,41**	**-9,42**	**-8,12**	**-21,04**	**-29,89**	**-25,52**	**-10,93**	**-26,13**	**-35,65**	**-26,80**	**-11,33**	**-29,68**	**-39,36**	**-33,00**	**-1,01**	**-2,85**	**-1,29**	**10,15**
F. P/E (Trailing)	-0,75	1,72	1,99	0,56	1,68	10,49	16,52	38,51	4,29	11,77	16,20	35,00	4,31	11,56	15,72	34,92	1,72	5,63	6,63	14,13
F. P/E (Forward)	**-5,04**	**-7,53**	**-9,45**	**4,83**	**1,98**	**6,66**	**11,59**	**25,74**	**4,84**	**14,95**	**21,60**	**30,96**	**4,14**	**14,78**	**22,75**	**34,68**	**4,41**	**12,77**	**19,21**	**27,52**
F. PEG (Trailing)	-2,88	0,17	2,24	-0,97	3,38	11,76	20,18	37,07	5,09	16,84	27,30	45,95	5,11	17,53	29,22	51,68	5,22	14,03	21,17	35,84
F. PEG (Forward)	**3,63**	**9,66**	**9,87**	**18,04**	**-0,99**	**2,45**	**2,77**	**15,88**	**-0,36**	**1,73**	**1,29**	**18,57**	**0,29**	**4,35**	**5,47**	**24,06**	**-1,24**	**-1,04**	**2,00**	**10,35**
F. P/B (Trailing)	1,62	8,25	9,20	13,25	0,78	6,46	7,13	26,73	0,85	5,73	7,81	31,96	2,27	9,45	12,73	39,27	-0,05	1,15	2,80	19,28
F. P/B (Forward)	**0,12**	**5,52**	**11,43**	**22,84**	**1,24**	**14,84**	**25,51**	**56,50**	**7,23**	**23,24**	**36,17**	**59,35**	**6,64**	**25,01**	**37,24**	**63,50**	**4,42**	**15,99**	**22,05**	**39,81**
F. EV/EBITDA Historical	4,28	7,06	8,05	4,65	14,76	30,17	35,43	31,12	15,96	37,82	48,50	47,08	15,05	34,64	42,11	47,35	11,09	23,13	26,01	35,71
F. EV/EBITDA Forward	**6,29**	**13,36**	**16,89**	**11,55**	**11,94**	**26,30**	**29,53**	**22,77**	**13,83**	**30,46**	**36,42**	**39,92**	**15,44**	**35,79**	**41,59**	**44,33**	**5,86**	**15,23**	**16,81**	**22,08**
F. Size	6,46	20,46	26,34	27,05	7,50	19,81	24,30	28,27	8,95	25,38	30,26	30,36	9,70	26,11	33,90	34,51	10,92	25,98	34,68	39,89

Table – B7 - EU. Long-short (F1-FN) spread return correlation for quintiles and top/bottom 20-30 stocks (only V/P)

	F1-FN Return Correlation																	
Factor	V/P (Quintile)	V/P (Top 20)	V/P (Top 30)	V/P (+5Y) (Quintile)	V/P (+5Y) (Top 20)	V/P (+5Y) (Top 30)	Target Price / P	ROE (Trailing)	ROE (Forward)	P/E (Trailing)	P/E (Forward)	PEG (Trailing)	PEG (Forward)	P/B (Trailing)	P/B (Forward)	EV/EBITDA Historical	EV/EBITDA Forward	Mkt Value Size
F. V/P (Quintile)	1,00																	
F. V/P (Top 20)	0,92	1,00																
F. V/P (Top 30)	0,95	0,98	1,00															
F. V/P (+5Y) (Quintile)	0,91	0,82	0,86	1,00														
F. V/P (+5Y) (Top 20)	0,55	0,59	0,58	0,65	1,00													
F. V/P (+5Y) (Top 30)	0,74	0,74	0,74	0,85	0,86	1,00												
F. Target Price / P	-0,23	-0,18	-0,19	-0,21	-0,10	-0,15	1,00											
F. ROE (Trailing)	0,20	0,17	0,19	0,10	-0,05	0,06	-0,43	1,00										
F. ROE (Forward)	0,02	0,00	0,01	0,27	0,28	0,26	0,22	-0,30	1,00									
F. P/E (Trailing)	-0,10	-0,07	-0,08	-0,24	-0,30	-0,23	-0,26	0,43	-0,57	1,00								
F. P/E (Forward)	0,28	0,25	0,28	0,17	-0,03	0,09	-0,35	0,46	-0,36	0,75	1,00							
F. PEG (Trailing)	0,58	0,51	0,56	0,44	0,19	0,32	-0,40	0,58	-0,36	0,48	0,82	1,00						
F. PEG (Forward)	0,13	0,14	0,16	0,01	-0,15	-0,06	-0,34	0,62	-0,47	0,74	0,73	0,62	1,00					
F. P/B (Trailing)	0,21	0,22	0,24	0,07	-0,12	-0,01	-0,34	0,58	-0,46	0,69	0,68	0,63	0,92	1,00				
F. P/B (Forward)	0,50	0,46	0,50	0,36	0,09	0,25	-0,32	0,44	-0,33	0,68	0,88	0,81	0,64	0,66	1,00			
F. EV/EBITDA Historical	0,61	0,48	0,53	0,52	0,27	0,41	-0,34	0,12	-0,24	0,12	0,39	0,55	0,28	0,31	0,54	1,00		
F. EV/EBITDA Forward	0,59	0,48	0,53	0,52	0,27	0,41	-0,27	0,02	-0,19	-0,06	0,22	0,41	0,16	0,21	0,37	0,81	1,00	
F. Size	0,01	-0,00	-0,00	0,05	-0,02	0,06	-0,01	0,05	0,03	-0,03	0,06	-0,01	-0,03	-0,03	0,01	0,07	0,14	1,00

APPENDIX C – US - Core Statistics on monthly rebalanced portfolios for sub-periods (95-05), (00-10), (07-10), (10-18)

Sub-period 1995-2005 (US)

		Universe Return						Sharpe Ratio					Beta				
Factor	F1-FN	1	2	3	4	5	Benchmark Return	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	14,85	18,03	15,63	16,56	7,41	-0,86	**9,28**	**1,15**	**1,10**	**1,03**	**0,36**	**-0,04**	**0,77**	**0,80**	**0,91**	**1,19**	**1,18**
F. V/P (Top 20)	23,31	24,27	11,95	-3,65			9,28	1,37	0,75	-0,14			0,77	0,96	1,24		
F. V/P (Top 30)	17,05	22,13	11,92	0,37			**9,28**	**1,31**	**0,74**	**0,02**			**0,76**	**0,97**	**1,17**		
F. V/P (+5Y) (Quintile)	9,46	16,00	15,65	15,93	7,41	2,11	9,28	1,07	1,09	1,04	0,39	0,09	0,80	0,79	0,90	1,09	1,22
F. V/P (+5Y) (Top 20)	13,67	19,06	11,97	0,83			**9,28**	**1,03**	**0,78**	**0,03**			**0,94**	**0,93**	**1,31**		
F. V/P (+5Y) (Top 30)	10,09	18,12	12,15	2,81			9,28	1,03	0,80	0,11			0,92	0,92	1,31		
F. Target Price / P	-11,19	6,33	11,63	15,13	13,85	17,76	**9,28**	**0,29**	**0,62**	**0,84**	**0,77**	**1,00**	**1,09**	**1,11**	**1,02**	**0,90**	**0,88**
F. ROE (Trailing)	-0,91	6,39	10,38	5,83	12,37	8,30	9,28	0,26	0,65	0,28	0,77	0,55	1,60	1,07	0,92	0,73	0,95
F. ROE (Forward)	8,96	14,44	14,19	13,75	10,39	1,25	**9,28**	**0,85**	**0,88**	**0,88**	**0,73**	**0,05**	**1,04**	**0,95**	**0,90**	**0,79**	**1,30**
F. P/E (Trailing)	-8,53	3,80	12,84	13,11	13,10	14,29	9,28	0,16	0,85	0,84	0,81	0,83	1,29	0,84	0,90	0,95	1,00
F. P/E (Forward)	-2,23	6,17	15,06	13,74	9,46	8,52	**9,28**	**0,24**	**1,00**	**0,93**	**0,61**	**0,35**	**1,27**	**0,73**	**0,83**	**0,95**	**1,21**
F. PEG (Trailing)	-0,34	10,99	12,69	12,01	11,69	7,89	9,28	0,57	0,84	0,80	0,74	0,29	1,03	0,76	0,86	0,95	1,36
F. PEG (Forward)	-1,06	6,09	14,47	15,71	11,23	7,42	**9,28**	**0,25**	**0,76**	**1,12**	**0,77**	**0,38**	**1,29**	**1,03**	**0,79**	**0,79**	**0,95**
F. P/B (Trailing)	4,16	9,68	16,23	13,73	10,28	4,56	9,28	0,42	0,91	0,89	0,65	0,22	1,22	0,96	0,87	0,84	0,94
F. P/B (Forward)	3,32	14,96	13,66	10,88	8,99	4,92	**9,28**	**0,86**	**0,88**	**0,64**	**0,50**	**0,18**	**0,90**	**0,87**	**0,99**	**1,04**	**1,19**
F. EV/EBITDA Historical	4,01	9,05	10,39	9,46	5,80	0,77	9,28	0,50	0,67	0,65	0,35	0,03	0,96	0,82	0,84	1,01	1,57
F. EV/EBITDA Forward	4,01	9,05	10,39	9,46	5,80	0,77	**9,28**	**0,50**	**0,67**	**0,65**	**0,35**	**0,03**	**0,96**	**0,82**	**0,84**	**1,01**	**1,57**
F. Mkt Value Size	4,43	10,34	15,61	9,72	8,64	5,70	9,28	0,56	0,64	0,59	0,52	0,37	1,01	1,14	0,92	0,96	0,92

Sub-period 2000-2010 (US)

		Universe Return						Sharpe Ratio					Beta				
Factor	F1-FN	1	2	3	4	5	Benchmark Return	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	8,34	6,67	7,78	6,90	2,60	-3,44	**1,64**	**0,38**	**0,51**	**0,39**	**0,13**	**-0,15**	**0,95**	**0,88**	**1,02**	**1,19**	**1,34**
F. V/P (Top 20)	20,49	9,34	4,82	-11,80			1,64	0,42	0,27	-0,43			1,11	1,05	1,54		
F. V/P (Top 30)	16,10	10,30	4,71	-7,17			**1,64**	**0,49**	**0,27**	**-0,28**			**1,08**	**1,04**	**1,47**		
F. V/P (+5Y) (Quintile)	8,19	7,09	6,26	7,45	3,78	-3,52	1,64	0,43	0,40	0,43	0,20	-0,14	0,92	0,90	1,03	1,12	1,40
F. V/P (+5Y) (Top 20)	13,08	5,67	5,11	-9,37			**1,64**	**0,31**	**0,29**	**-0,36**			**0,97**	**1,06**	**1,42**		
F. V/P (+5Y) (Top 30)	10,98	5,02	5,45	-8,14			1,64	0,28	0,31	-0,31			0,96	1,04	1,46		
F. Target Price / P	-5,67	0,48	4,10	6,06	7,34	4,75	**1,64**	**0,03**	**0,22**	**0,34**	**0,40**	**0,22**	**1,02**	**1,08**	**1,05**	**1,05**	**1,24**
F. ROE (Trailing)	-0,59	3,45	7,60	6,05	8,16	5,05	1,64	0,13	0,42	0,35	0,48	0,29	1,50	1,10	0,91	0,89	1,00
F. ROE (Forward)	10,21	10,38	6,28	5,94	4,89	-1,09	**1,64**	**0,34**	**0,37**	**0,35**	**0,28**	**-0,05**	**1,29**	**1,00**	**0,99**	**1,01**	**1,38**
F. P/E (Trailing)	-4,87	-1,12	5,82	7,07	6,13	4,82	1,64	-0,05	0,33	0,42	0,36	0,27	1,44	1,04	0,99	0,98	1,00
F. P/E (Forward)	5,62	6,08	8,62	7,11	3,12	1,21	**1,64**	**0,17**	**0,51**	**0,45**	**0,20**	**0,06**	**1,63**	**0,97**	**0,92**	**0,93**	**1,13**
F. PEG (Trailing)	2,37	3,09	7,69	7,23	4,03	-0,20	1,64	0,14	0,46	0,47	0,24	-0,01	1,28	0,95	0,91	0,99	1,32
F. PEG (Forward)	-2,48	-1,20	6,97	6,93	2,36	2,58	**1,64**	**-0,05**	**0,31**	**0,43**	**0,16**	**0,15**	**1,51**	**1,31**	**0,95**	**0,87**	**0,95**
F. P/B (Trailing)	0,87	0,78	7,77	4,58	2,90	1,03	1,64	0,03	0,37	0,27	0,19	0,06	1,52	1,22	1,00	0,89	0,97
F. P/B (Forward)	9,59	8,78	8,39	6,30	3,92	-0,95	**1,64**	**0,27**	**0,48**	**0,35**	**0,21**	**-0,05**	**1,41**	**1,01**	**1,04**	**1,07**	**1,04**
F. EV/EBITDA Historical	7,29	8,60	7,60	4,68	4,93	-0,98	1,64	0,42	0,45	0,29	0,28	-0,04	1,12	0,95	0,92	1,04	1,46
F. EV/EBITDA Forward	7,29	8,60	7,60	4,68	4,93	-0,98	**1,64**	**0,42**	**0,45**	**0,29**	**0,28**	**-0,04**	**1,12**	**0,95**	**0,92**	**1,04**	**1,46**
F. Mkt Value Size	5,69	5,22	9,95	5,64	8,47	-0,03	1,64	0,27	0,33	0,30	0,28	-0,00	1,13	1,28	1,09	1,26	0,93

Sub-period 2007-2010 (US)

		Universe Return						Sharpe Ratio					Beta				
Factor	F1-FN	1	2	3	4	5	Benchmark Return	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	-1,18	-3,12	2,82	2,82	0,38	-3,61	**-2,04**	**-0,12**	**0,13**	**0,12**	**0,01**	**-0,12**	**1,11**	**0,95**	**1,06**	**1,13**	**1,32**
F. V/P (Top 20)	5,28	-6,02	0,65	-13,21			-2,04	-0,18	0,03	-0,36			1,33	1,09	1,54		
F. V/P (Top 30)	3,89	-2,51	0,54	-8,78			**-2,04**	**-0,08**	**0,02**	**-0,25**			**1,27**	**1,08**	**1,51**		
F. V/P (+5Y) (Quintile)	5,53	2,13	1,30	1,28	0,68	-5,85	-2,04	0,09	0,06	0,05	0,03	-0,19	1,02	1,02	1,09	1,09	1,36
F. V/P (+5Y) (Top 20)	7,79	-1,64	0,54	-12,39			**-2,04**	**-0,07**	**0,02**	**-0,38**			**0,97**	**1,11**	**1,34**		
F. V/P (+5Y) (Top 30)	5,56	-3,46	0,94	-11,95			-2,04	-0,14	0,04	-0,36			1,00	1,10	1,42		
F. Target Price / P	0,16	0,05	1,72	1,54	-1,15	-4,11	**-2,04**	**0,00**	**0,07**	**0,06**	**-0,04**	**-0,13**	**0,94**	**1,04**	**1,08**	**1,16**	**1,38**
F. ROE (Trailing)	4,80	-0,72	1,81	1,26	-0,71	-3,69	-2,04	-0,02	0,07	0,06	-0,03	-0,16	1,46	1,13	1,00	1,00	1,02
F. ROE (Forward)	24,94	19,54	2,95	-1,72	-4,19	-4,10	**-2,04**	**0,38**	**0,13**	**-0,07**	**-0,16**	**-0,14**	**1,51**	**1,03**	**1,06**	**1,14**	**1,28**
F. P/E (Trailing)	-1,69	-3,59	-0,78	3,02	1,25	-0,75	-2,04	-0,11	-0,03	0,13	0,05	-0,03	1,39	1,10	1,06	1,02	1,02
F. P/E (Forward)	15,49	13,43	0,52	-0,91	-2,29	0,30	**-2,04**	**0,25**	**0,02**	**-0,04**	**-0,11**	**0,01**	**1,71**	**1,15**	**1,01**	**0,96**	**1,01**
F. PEG (Trailing)	-2,81	-4,99	1,65	1,53	1,47	-1,25	-2,04	-0,16	0,07	0,07	0,06	-0,05	1,37	1,13	0,95	1,01	1,16
F. PEG (Forward)	-4,08	-4,98	0,16	3,70	-2,32	1,39	**-2,04**	**-0,15**	**0,01**	**0,16**	**-0,12**	**0,07**	**1,43**	**1,40**	**1,04**	**0,89**	**0,86**
F. P/B (Trailing)	-5,31	-7,25	3,18	-0,13	1,17	0,66	-2,04	-0,21	0,11	-0,01	0,06	0,03	1,51	1,31	1,03	0,90	0,87
F. P/B (Forward)	16,27	12,13	-1,18	1,31	0,70	-2,08	**-2,04**	**0,23**	**-0,05**	**0,05**	**0,03**	**-0,09**	**1,66**	**1,10**	**1,07**	**1,05**	**0,95**
F. EV/EBITDA Historical	-6,51	-2,40	1,20	-2,55	3,79	3,19	-2,04	-0,08	0,05	-0,11	0,17	0,11	1,19	1,04	1,00	1,02	1,32
F. EV/EBITDA Forward	-6,51	-2,40	1,20	-2,55	3,79	3,19	**-2,04**	**-0,08**	**0,05**	**-0,11**	**0,17**	**0,11**	**1,19**	**1,04**	**1,00**	**1,02**	**1,32**
F. Mkt Value Size	4,23	0,06	15,46	0,34	17,00	-3,10	-2,04	0,00	0,32	0,01	0,34	-0,15	1,15	1,45	1,17	1,47	0,94

Sub-period 2010-2018 (US)

		Universe Return						Sharpe Ratio					Beta				
Factor	F1-FN	1	2	3	4	5	Benchmark Return	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	3,77	14,95	11,87	11,86	10,64	10,08	**12,26**	**1,28**	**0,96**	**0,92**	**0,80**	**0,69**	**0,95**	**1,03**	**1,09**	**1,10**	**1,15**
F. V/P (Top 20)	3,55	15,61	11,84	9,49			12,26	1,18	0,95	0,48			0,94	1,06	1,36		
F. V/P (Top 30)	1,75	15,03	11,74	11,34			**12,26**	**1,19**	**0,94**	**0,63**			**0,95**	**1,06**	**1,29**		
F. V/P (+5Y) (Quintile)	1,79	14,04	12,71	11,49	10,01	11,49	12,26	1,17	1,07	0,92	0,73	0,79	0,99	0,99	1,04	1,15	1,15
F. V/P (+5Y) (Top 20)	-2,34	12,16	11,90	13,71			**12,26**	**0,87**	**0,95**	**0,79**			**1,10**	**1,05**	**1,28**		
F. V/P (+5Y) (Top 30)	0,36	13,70	11,84	12,30			12,26	1,02	0,95	0,74			1,06	1,05	1,24		
F. Target Price / P	2,61	14,38	11,29	10,96	11,49	10,90	**12,26**	**1,10**	**0,91**	**0,87**	**0,92**	**0,75**	**1,04**	**1,03**	**1,06**	**1,02**	**1,17**
F. ROE (Trailing)	0,45	10,88	14,15	11,52	11,55	10,84	12,26	0,60	0,98	0,94	1,10	1,07	1,43	1,20	1,03	0,87	0,79
F. ROE (Forward)	-0,64	11,29	13,10	11,47	11,01	11,44	**12,26**	**0,92**	**1,03**	**0,91**	**0,91**	**0,79**	**1,02**	**1,06**	**1,06**	**1,00**	**1,13**
F. P/E (Trailing)	-2,16	10,80	9,99	12,64	12,71	13,15	12,26	0,78	0,82	1,00	0,99	1,00	1,10	1,00	1,06	1,08	1,08
F. P/E (Forward)	-1,84	10,39	11,57	12,24	11,74	12,58	**12,26**	**0,66**	**0,88**	**1,04**	**0,99**	**0,99**	**1,24**	**1,08**	**0,98**	**0,98**	**1,01**
F. PEG (Trailing)	0,43	11,20	11,58	14,42	11,01	10,91	12,26	0,68	0,89	1,29	0,93	0,85	1,32	1,08	0,93	0,97	1,01
F. PEG (Forward)	-2,65	9,38	11,24	12,60	13,19	12,58	**12,26**	**0,61**	**0,74**	**1,00**	**1,15**	**1,16**	**1,22**	**1,26**	**1,05**	**0,95**	**0,86**
F. P/B (Trailing)	-3,03	8,05	12,86	13,30	13,04	11,73	12,26	0,49	0,85	1,06	1,13	1,17	1,31	1,24	1,05	0,95	0,77
F. P/B (Forward)	-4,83	9,38	11,54	11,13	11,52	14,50	**12,26**	**0,65**	**0,93**	**0,88**	**0,95**	**1,05**	**1,16**	**1,01**	**1,05**	**1,01**	**1,05**
F. EV/EBITDA Historical	-2,71	9,92	12,06	13,16	12,48	12,93	12,26	0,67	0,98	1,12	1,06	0,97	1,20	1,03	0,98	0,99	1,06
F. EV/EBITDA Forward	-2,71	9,92	12,06	13,16	12,48	12,93	**12,26**	**0,67**	**0,98**	**1,12**	**1,06**	**0,97**	**1,20**	**1,03**	**0,98**	**0,99**	**1,06**
F. Mkt Value Size	2,80	14,71	9,58	10,92	10,74	11,79	12,26	1,04	0,74	0,86	0,89	1,01	1,13	1,08	1,05	1,01	1,00

APPENDIX D - Europe - Core Statistics on monthly rebalanced portfolios for sub-periods (95-05), (00-10), (07-10), (10-18)

Sub-period 1995-2005 (EU)

		Universe Return						Sharpe Ratio					Beta				
Factor	F1-FN	1	2	3	4	5	Benchmark Return	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	10,63	20,68	14,03	11,26	9,04	5,81	**8,45**	**1,32**	**0,92**	**0,74**	**0,49**	**0,25**	**0,74**	**0,81**	**0,84**	**1,06**	**1,22**
F. V/P (Top 20)	11,74	20,45	12,31	3,71			8,45	1,19	0,76	0,15			0,74	0,93	1,20		
F. V/P (Top 30)	11,54	20,96	12,23	4,33			**8,45**	**1,25**	**0,75**	**0,18**			**0,72**	**0,93**	**1,19**		
F. V/P (+5Y) (Quintile)	5,73	16,95	14,05	12,95	9,35	8,23	8,45	1,12	0,96	0,83	0,50	0,38	0,78	0,77	0,88	1,07	1,16
F. V/P (+5Y) (Top 20)	0,12	11,40	12,68	7,65			**8,45**	**0,66**	**0,79**	**0,31**			**0,79**	**0,93**	**1,22**		
F. V/P (+5Y) (Top 30)	4,20	12,72	12,94	5,29			8,45	0,76	0,81	0,22			0,84	0,92	1,24		
F. Target Price / P	-10,57	7,79	10,16	11,46	14,05	19,23	**8,45**	**0,36**	**0,64**	**0,72**	**0,85**	**1,02**	**1,12**	**0,89**	**0,89**	**0,89**	**0,96**
F. ROE (Trailing)	4,19	3,74	8,93	9,46	4,36	-0,41	8,45	0,13	0,49	0,57	0,27	-0,02	1,42	1,02	0,84	0,75	0,94
F. ROE (Forward)	-0,92	10,81	14,66	13,04	13,47	11,31	**8,45**	**0,65**	**0,94**	**0,85**	**0,83**	**0,62**	**0,92**	**0,87**	**0,85**	**0,92**	**1,02**
F. P/E (Trailing)	-3,01	9,26	14,02	13,03	12,79	13,17	8,45	0,45	0,83	0,81	0,86	0,81	1,14	0,95	0,91	0,83	0,90
F. P/E (Forward)	3,59	12,97	16,96	12,81	11,35	8,53	**8,45**	**0,64**	**1,13**	**0,93**	**0,76**	**0,42**	**1,12**	**0,78**	**0,74**	**0,86**	**1,09**
F. PEG (Trailing)	11,88	19,61	13,52	12,55	11,10	5,42	8,45	1,01	0,89	0,86	0,71	0,24	1,05	0,79	0,80	0,91	1,19
F. PEG (Forward)	4,22	12,42	16,51	13,58	11,23	8,16	**8,45**	**0,60**	**0,88**	**0,86**	**0,78**	**0,49**	**1,14**	**1,03**	**0,88**	**0,81**	**0,91**
F. P/B (Trailing)	7,47	15,89	15,12	13,25	9,74	8,17	8,45	0,75	0,81	0,83	0,65	0,52	1,15	1,04	0,90	0,84	0,85
F. P/B (Forward)	7,77	18,20	14,37	11,07	11,13	7,87	**8,45**	**0,98**	**0,89**	**0,73**	**0,75**	**0,38**	**0,99**	**0,87**	**0,84**	**0,83**	**1,05**
F. EV/EBITDA Historical	6,99	15,65	14,37	12,59	10,12	6,58	8,45	0,93	0,99	0,90	0,64	0,32	0,91	0,76	0,76	0,89	1,10
F. EV/EBITDA Forward	6,87	12,92	11,57	7,42	1,50	3,25	**8,45**	**0,76**	**0,81**	**0,47**	**0,09**	**0,14**	**0,89**	**0,74**	**0,84**	**0,98**	**1,31**
F. Size	0,68	12,20	13,21	13,91	12,98	10,82	8,45	0,77	0,82	0,87	0,79	0,61	0,86	0,88	0,89	0,95	1,04

Sub-period 2000-2010 (EU)

		Universe Return						Sharpe Ratio					Beta				
Factor	F1-FN	1	2	3	4	5	Benchmark Return	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	8,92	9,22	5,78	4,93	2,95	-0,95	-2,45	0,48	0,33	0,28	0,15	-0,04	1,02	0,98	1,00	1,10	1,25
F. V/P (Top 20)	8,00	7,63	4,57	-2,12			-2,45	0,34	0,25	-0,09			1,11	1,06	1,23		
F. V/P (Top 30)	7,88	8,05	4,56	-1,47			-2,45	0,38	0,25	-0,06			1,06	1,06	1,25		
F. V/P (+5Y) (Quintile)	6,43	7,31	7,18	5,16	2,50	-0,27	-2,45	0,39	0,41	0,29	0,13	-0,01	1,03	0,97	1,02	1,09	1,25
F. V/P (+5Y) (Top 20)	6,31	6,79	4,60	-2,31			-2,45	0,36	0,25	-0,09			0,95	1,07	1,31		
F. V/P (+5Y) (Top 30)	7,78	7,42	4,68	-2,34			-2,45	0,39	0,25	-0,10			1,02	1,06	1,33		
F. Target Price / P	-8,57	0,79	4,98	5,37	2,95	7,38	-2,45	0,04	0,31	0,31	0,15	0,29	0,99	0,91	0,99	1,09	1,37
F. ROE (Trailing)	2,21	2,52	5,33	6,58	5,27	0,89	-2,45	0,09	0,27	0,39	0,32	0,05	1,42	1,10	0,93	0,87	0,94
F. ROE (Forward)	1,26	4,84	5,67	5,73	4,41	3,00	-2,45	0,28	0,31	0,32	0,24	0,16	0,96	1,02	1,02	1,05	1,08
F. P/E (Trailing)	-2,14	2,40	3,26	5,35	5,61	5,43	-2,45	0,10	0,17	0,29	0,33	0,32	1,33	1,11	1,02	0,95	0,92
F. P/E (Forward)	4,09	3,79	8,30	6,93	3,76	0,52	-2,45	0,16	0,44	0,44	0,24	0,03	1,36	1,03	0,88	0,88	0,97
F. PEG (Trailing)	9,02	6,61	6,13	6,56	4,42	-1,73	-2,45	0,27	0,34	0,40	0,27	-0,09	1,37	1,00	0,92	0,93	1,11
F. PEG (Forward)	1,17	3,95	4,82	3,63	5,52	3,69	-2,45	0,17	0,21	0,20	0,37	0,23	1,31	1,30	1,04	0,83	0,90
F. P/B (Trailing)	1,23	4,62	4,70	3,52	4,10	4,69	-2,45	0,18	0,21	0,20	0,26	0,30	1,42	1,25	0,98	0,87	0,86
F. P/B (Forward)	4,90	6,32	3,96	5,62	5,31	2,00	-2,45	0,26	0,21	0,33	0,35	0,12	1,35	1,06	0,97	0,84	0,89
F. EV/EBITDA Historical	11,57	9,87	9,32	8,82	3,67	-2,01	-2,45	0,53	0,54	0,55	0,22	-0,10	1,04	0,95	0,90	0,94	1,07
F. EV/EBITDA Forward	8,42	9,62	8,48	6,93	2,02	0,19	-2,45	0,49	0,50	0,42	0,11	0,01	1,05	0,94	0,94	0,99	1,23
F. Size	4,37	5,79	6,21	6,41	3,81	1,19	-2,45	0,32	0,34	0,35	0,21	0,07	1,02	1,00	1,04	1,04	1,04

Sub-period 2007-2010 (EU)

		Universe Return						Sharpe Ratio					Beta				
Factor	F1-FN	1	2	3	4	5	Benchmark Return	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	-2,34	-7,34	-1,87	-0,58	1,12	-5,13	-8,20	-0,28	-0,08	-0,02	0,05	-0,21	1,17	1,04	1,09	1,08	1,16
F. V/P (Top 20)	-0,21	-9,86	-2,12	-9,39			-8,20	-0,32	-0,09	-0,36			1,34	1,10	1,15		
F. V/P (Top 30)	0,77	-9,60	-1,73	-10,41			-8,20	-0,34	-0,07	-0,41			1,23	1,10	1,17		
F. V/P (+5Y) (Quintile)	-0,03	-5,41	-1,10	-3,08	1,33	-5,69	-8,20	-0,21	-0,05	-0,13	0,06	-0,22	1,16	1,09	1,07	1,03	1,20
F. V/P (+5Y) (Top 20)	9,14	0,92	-2,67	-8,68			-8,20	0,04	-0,11	-0,33			1,03	1,11	1,16		
F. V/P (+5Y) (Top 30)	8,62	0,82	-2,64	-8,01			-8,20	0,03	-0,11	-0,30			1,11	1,10	1,22		
F. Target Price / P	-2,51	-0,70	-0,13	-2,26	-6,58	-4,22	-8,20	-0,04	-0,01	-0,10	-0,26	-0,12	0,86	0,90	1,04	1,17	1,55
F. ROE (Trailing)	4,92	-2,84	-3,49	-1,27	-0,89	-5,66	-8,20	-0,09	-0,14	-0,06	-0,04	-0,28	1,45	1,20	1,00	0,94	0,93
F. ROE (Forward)	5,32	2,73	-3,34	-2,85	-3,98	-3,11	-8,20	0,13	-0,14	-0,13	-0,16	-0,14	0,95	1,09	1,06	1,12	1,06
F. P/E (Trailing)	-1,66	-3,81	-4,91	-2,26	-1,09	-1,20	-8,20	-0,13	-0,20	-0,10	-0,05	-0,06	1,35	1,18	1,05	1,01	0,92
F. P/E (Forward)	0,18	-3,41	-1,03	-2,28	-2,51	-2,05	-8,20	-0,11	-0,04	-0,11	-0,14	-0,11	1,41	1,18	1,00	0,85	0,84
F. PEG (Trailing)	-1,31	-7,37	-3,15	0,60	1,28	-4,55	-8,20	-0,23	-0,13	0,03	0,07	-0,21	1,48	1,10	1,00	0,90	1,01
F. PEG (Forward)	0,38	-2,07	-4,17	-5,34	-0,93	-1,05	-8,20	-0,08	-0,13	-0,22	-0,05	-0,06	1,25	1,44	1,15	0,84	0,85
F. P/B (Trailing)	-2,08	-5,65	-1,83	-4,33	-0,21	-1,66	-8,20	-0,18	-0,06	-0,20	-0,01	-0,09	1,40	1,40	1,01	0,84	0,88
F. P/B (Forward)	-1,76	-3,70	-6,32	-0,94	-0,64	-0,32	-8,20	-0,11	-0,26	-0,04	-0,03	-0,02	1,51	1,13	1,02	0,85	0,79
F. EV/EBITDA Historical	7,77	1,91	0,76	4,12	-0,82	-5,23	-8,20	0,08	0,03	0,19	-0,04	-0,24	1,08	1,04	0,97	0,87	0,98
F. EV/EBITDA Forward	-0,13	-2,62	2,67	2,15	-0,79	-2,64	-8,20	-0,11	0,12	0,10	-0,04	-0,11	1,11	1,04	0,99	0,92	1,11
F. Size	5,26	-0,40	-0,94	-0,01	-4,54	-5,27	-8,20	-0,02	-0,04	-0,00	-0,20	-0,24	1,07	1,04	1,09	1,09	1,02

Sub-period 2010-2018 (EU)

		Universe Return						Sharpe Ratio					Beta				
Factor	F1-FN	1	2	3	4	5	Benchmark Return	1	2	3	4	5	1	2	3	4	5
F. V/P (Quintile)	1,95	9,11	9,03	8,34	6,41	6,74	3,30	0,75	0,73	0,69	0,52	0,53	0,91	0,96	0,95	0,95	0,94
F. V/P (Top 20)	1,93	9,00	7,97	6,05			3,30	0,59	0,67	0,38			0,97	0,94	1,04		
F. V/P (Top 30)	0,82	8,71	7,94	7,17			3,30	0,61	0,66	0,48			0,97	0,94	0,98		
F. V/P (+5Y) (Quintile)	2,40	10,00	8,11	7,69	7,21	6,78	3,30	0,93	0,68	0,63	0,56	0,48	0,81	0,93	0,95	1,00	1,04
F. V/P (+5Y) (Top 20)	3,44	8,93	8,11	3,53			3,30	0,72	0,68	0,19			0,82	0,94	1,21		
F. V/P (+5Y) (Top 30)	1,23	8,77	8,03	6,26			3,30	0,74	0,67	0,38			0,82	0,94	1,11		
F. Target Price / P	4,12	11,32	8,97	7,82	6,92	5,14	3,30	1,08	0,79	0,64	0,54	0,30	0,73	0,86	0,94	0,99	1,21
F. ROE (Trailing)	-1,33	5,88	8,51	8,56	9,26	7,78	3,30	0,33	0,63	0,75	0,90	0,82	1,30	1,05	0,89	0,79	0,69
F. ROE (Forward)	1,73	8,72	8,54	8,44	8,82	5,97	3,30	0,84	0,72	0,72	0,71	0,40	0,77	0,91	0,91	0,96	1,10
F. P/E (Trailing)	-4,47	5,21	5,77	8,89	9,94	10,18	3,30	0,35	0,46	0,73	0,83	0,92	1,08	0,97	0,95	0,92	0,81
F. P/E (Forward)	-1,51	7,17	6,67	7,93	9,74	8,93	3,30	0,49	0,52	0,67	0,90	0,78	1,07	0,99	0,91	0,82	0,85
F. PEG (Trailing)	-0,72	7,26	7,11	8,12	8,88	8,38	3,30	0,42	0,55	0,71	0,83	0,78	1,27	0,99	0,88	0,81	0,78
F. PEG (Forward)	-4,31	4,07	7,06	9,13	10,60	9,04	3,30	0,27	0,50	0,77	0,96	0,88	1,10	1,08	0,92	0,83	0,79
F. P/B (Trailing)	-5,01	2,95	8,30	8,89	10,89	8,75	3,30	0,18	0,60	0,75	1,03	0,88	1,19	1,06	0,92	0,80	0,76
F. P/B (Forward)	-4,32	5,78	6,93	7,54	9,40	10,56	3,30	0,36	0,55	0,63	0,83	1,01	1,16	0,98	0,92	0,86	0,73
F. EV/EBITDA Historical	0,04	8,88	6,65	9,38	9,84	8,92	3,30	0,65	0,52	0,87	0,92	0,80	1,03	1,00	0,83	0,78	0,81
F. EV/EBITDA Forward	-1,96	8,64	7,29	8,07	9,87	10,97	3,30	0,60	0,59	0,71	0,91	1,00	1,08	0,96	0,88	0,81	0,80
F. Size	2,31	8,76	9,58	7,82	8,40	6,24	3,30	0,70	0,81	0,67	0,67	0,52	0,94	0,88	0,90	0,98	0,96

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Critical issues when valuing small businesses

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Small and medium enterprises (SMEs) are a crucial component of local and global economies. Their wealth can support or hinder the sustainable development of many countries. The appropriate determination of the transaction values of smaller businesses represents an important variable in the functioning of the markets. Notwithstanding, best practices of business valuations often refer to the ongoing conditions of large, public companies. This paper investigates the main criticalities that a practitioner may have to deal with when valuing SMEs by applying generally recognized methods. The last part of the paper considers the potential adoption of generally recognized standards for SMEs.

1. Introduction

Smaller businesses are a fundamental component of local and global economies. Research estimates indicate that small and medium enterprises (SMEs) represent more than 95% of the world's enterprises and account for approximately 60% of private sector employment (Edinburgh Group, 2012). In OECD countries, SMEs create around 60% of total employment and from 50% to 60% of value added (OECD, 2017). Smaller enterprises are crucial to the financial health and a sustainable economic growth of both developed and developing countries.

This paper is dedicated to the analysis of the peculiarities arising in the valuation of an SME. Despite the economy data, valuation methods proposed by practitioners are not focussed on the SME as the target entity.

The general idea could be that the valuation of an SME may be easier than the valuation of a public company. Paradoxically, it could be the opposite. Best practices are often built referring to the valuation of large, public companies, that usually have specific safeguards to avoid the mix between personal and business affairs, operate in "perfect" financial markets and provide a high level of information to their stakeholders, and specifically to the financial community.

Understanding SMEs' value becomes crucial for their stakeholders, such as the ownership that has to decide whether it is more appropriate to succeed or to sell, cease or continue its operations by combining with other enterprises or entering financial markets, also considering the minority interests (when they exist), and the employees who may aspire to take over the business.

The paper approaches the issues by a qualitative perspective. As a matter of fact, the valuation of an SME cannot be easily investigated by a quantitative point of view, for a number of reasons, including the following: transactions are usually not public, as they are operated in non-regulated markets; the universe of the operations is not determinable; and, each deal can present very peculiar features.

This document illustrates the difficulties that practitioners may face when valuing smaller businesses - without investigating the peculiar issues related to the valuation of interests in SMEs-, especially when they apply generally recognized practices. Aiming at providing an overview of the main criticalities, the research contextualizes the applied concept of SME, then investigates -functionally to the purposes of this paper- the main qualitative characteristics of smaller businesses and the main issues a practitioner can experience when adopting the consolidated practices. Lastly, the authors express some considerations about the approach that could be adopted in the valuation of SMEs.

2. The "concept" of small and medium enterprise

2.1. Regulatory approach

As of today, it is not possible to give a univocal and generally accepted definition of SME. Each definition needs to be placed in the context in which it arises in order to fully understand the objective it is intended to pursue.

The approach to the definition of SME varies depending on its "location" and application. This last feature is especially clear when the definitions established by regulators (regulatory approach) are opposed to the ones provided by the technical bodies (professional approach). From this point of view, the para-

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graph illustrates some significant cases emerging from local and regional jurisdictions and standard setters.

Regulators usually apply a quantitative approach to identify SMEs (Dennis, 1982; Jarvis *et al.*, 2000), as definitions must be objectively determined in order to clarify the scope of application and the recipients of the relevant rules. The predominant feature which identifies an enterprise is the size. In many legal environments, businesses can be categorized in: large enterprises, medium enterprises, small enterprises, and micro-enterprises.

At the same time, economies have a different understanding of the size of an enterprise. The U. S. Small Business Administration provides a "Table of Small Business Size Standards", where it categorizes enterprises in relation to the NAICS Industry Description. This classification is based on average annual income or the average number of employees of a business. To have an idea of the required size, most categories classify an enterprise as a small enterprise when it has less than 500 employees.

The European Union defines the categories of enterprises based on three thresholds: turnover; total assets; and average number of employees. Specifically, the 2013/34/EU, based on the "think small first" approach, states that undertakings are categorized considering if on their balance sheet date, they do not exceed (or exceed) the limits of at least two of the three criteria exposed in the following table.

Micro undertakings (up to)	Small undertakings (up to)	Medium undertakings (up to)	Large undertakings (over)
(a) balance sheet total: EUR 350 000; (b) net turnover: EUR 700 000; (c) average number of employees during the financial year: 10	(a) balance sheet total: EUR 4000000; (b) net turnover: EUR 8000000; (c) average number of employees during the financial year: 50.	(a) balance sheet total: EUR 20 000 000; (b) net turnover: EUR 40 000 000; (c) average number of employees during the financial year: 250.	(a) balance sheet total: EUR 20 000 000; (b) net turnover: EUR 40 000 000; (c) average number of employees during the financial year: 250.

Member States may define thresholds exceeding the thresholds in points (a) and (b) of small undertakings. However, the thresholds should not exceed EUR 6.000.000 for the balance sheet total and EUR 12.000.000 for the net turnover.

The Indian government recently approved a new classification of enterprises based on annual revenue and replacing the former definition based on investment in tangible assets (plant and machinery). In this perspective:

a. micro enterprises present total annual revenue up to Rs 5 crore (approximately € 600.000);

b. small enterprises have total annual revenue from Rs 5 crore to Rs 75 crore (approximately from € 600.000 to € 8.970.000);

c. medium enterprises have total annual revenue from Rs 75 crore to Rs 250 crore (approximately from € 8.970.000 to € 30.500.000);

d. large enterprises have total annual revenue exceeding Rs 250 crore (exceeding approximately € 30.500.000).

As previously mentioned and then illustrated, definitions can vary in relation to the pursued aims, economy and cultural organization.

2.2. Professional approach

From a technical and professional point of view, the size of an enterprise is often combined –if not replaced- with other qualitative factors. The focus is essentially on the appropriateness of the applied standards.

As far as accounting standards are concerned, IFRS for SMEs is intended to be used by SMEs, which are entities that publish general purpose financial statements for external users and do not have public accountability. Specifically, "An entity has public accountability under the IASB's definition if it files, or is in the process of filing, its financial statements with a securities commission or other regulatory organization for the purpose of issuing any class of instruments in a public market; or it holds assets in a fiduciary capacity for a broad group of outsiders. Examples of entities that hold assets in a fiduciary capacity include banks, insurance companies, brokers and dealers in securities, pension funds and mutual funds" (IASB, 2015).

The issue of the adoption of professional standards by SMEs has been dealt with by the International Audit and Assurance Standards Board (IAASB), the most authoritative audit standard setter. IAASB enacted the International Auditing Practice Statement (IAPS) 1005 (IAASB, 2002), "Special Considerations in the Audit of Small Entities". IAPS 1005 has then been withdrawn as a result of the Clarity project and its content has been included, where appropriate, in the relevant standards. However, IAASB provides, in relation to the special considerations, the following list of "… qualitative characteristics, such as:

a. Concentration of ownership and management in a small number of individuals (often a single individual – either a natural person or another enterprise that owns the entity provided the owner exhibits the relevant qualitative characteristics); and

b. One or more of the following are also found:

(i) Straightforward or uncomplicated transactions;

(ii) Simple record-keeping;

(iii) Few lines of business and few products within business lines;
(iv) Few internal controls;
(v) Few levels of management with responsibility for a broad range of controls; or
(vi) Few personnel, many having a wide range of duties" (IAASB, 2016).

IAASB adds that the above defined qualitative characteristics are not exhaustive and non-inclusive to smaller entities; they do not have to be all existing to identify a "smaller entity" as well.

The International Valuation Standards Council (IVSC), as the most authoritative international organization in the valuation field, does not present special issues in relation to the valuation of SMEs, or a definition of smaller enterprises (IVSC, 2017).

The SME considered for the purpose of this research is:

- a for-profit entity. Technical experts sometimes assimilate small enterprises to not-for-profit organisations. The considerations of this paper only relate to entities oriented to profit;
- an enterprise which is not listed and does not aim to list its financial instruments in a regulated market. It is substantially a private company whose ambition is not to become public, or to turn to the financial markets to obtain resources;
- an enterprise that is not extremely complex in its management and has potential comparable entities in the market. However, this paper does not examine the case of micro-enterprises. Micros require further specific considerations. On the other side, "bigger" medium companies, apart from the territorial collocation, could have more similarities with large companies than with small businesses;
- an enterprise that does not belong to public company groups. SMEs belonging to groups follow different decisional approaches, are often managed and accounted for as a branch of the large company rather than as an "individual" SME.

Lastly, it has to be observed that SMEs are often family businesses. Issues and criticalities can be identical in many cases (Ballwieser, 2017). That said, the paper takes into consideration smaller entities regardless of the fact that they are family businesses.

3. Literature review

Academia, professional bodies and practitioners have addressed the matter of the valuation of SMEs, approaching this issue from different but, sometimes, overlapping perspectives.

With reference to the scope identification, SMEs have often been associated to closely held businesses (Dukes *et al.*, 1996), as a corporation whose owners are limited in numbers. From this perspective, the studies -moving from the fact that entities are not on open markets- focus more on the impact that the illiquidity of the stocks can have on deals. In other cases, small businesses are associated, as already mentioned, to family businesses; the small size is sometimes considered as a characteristic of family businesses (Ballwieser, 2017). The "size effect" is studied as an autonomous variable as well (Banz, 1981).

Researchers have suggested specific valuation criteria (Sridharan, 2012). Boudreaux *et al.* (2011) propose to value business units by discounting cash flows with a discount rate reflecting the stockholders' risk, usually higher than in public firms. Feldman (2005) proposes selected adjustments for the SMEs discounted cash flows, impacting on the determination of the specific variables.

Some scholars focus on the determination of specific variables in the SMEs context in the application of the Discounted Cash Flow method (DCF). The analyses start from the consideration that cost of capital is usually lower for a public company than for a private company (OIV, 2015). Accordingly, a body of literature has, for example, considered the fair measurement of the systematic risk for smaller enterprises (Damodaran, 2005).

Another body of literature identifies net assets value as an applicable method for business valuations, once the appropriate accounting data are adjusted in order to reflect their current value (Liberatore, 2010).

Professional organizations usually adapt "original" standards to the qualities of smaller entities. Among the existing examples, in 2001 FEE (now ACE), the European Federation of Accountants, published some recommendations on how to approach the valuation of smaller entities. The paper investigates the different conditions that could "deviate" the adoption of usual rules.

AECA (2005), the professional body operating in the enactment of good practices in business management in Spain, published a text dedicated to the valuation of SMEs, explaining and considering the adoption of DCF and net asset value methods.

At the same time, IDW (2014), the Institute of Public Auditors in Germany, approached more specifically the topic by publishing application guidelines, investigating the potential impact of the size and the characteristics of smaller businesses on valuations.

AICPA (2016) has enacted a guide on the valuation of privately-held-company equity securities when issued as compensation, as a consequence to what required by the US GAAP.

Professionals are certainly very concerned about the investigated topic, which is of extreme interest also for professional practices, given its relevance from the fiscal point of view. An example is in the US the enactment of the Revenue Ruling 59-60.

Some professionals apply this approach as well, determining the criteria that are capable of adjusting the standard measurements (Pratt et *al.*, 1996; Feldman, 2005; Trugman, 2017). Some researches intend to verify the effect of SMEs characteristics in determining price transactions in terms of the entity's size (Trugman G. and Trugman L., 2011).

4. The significant SMEs features in relation to business valuation

4.1. Structural features

SMEs often present some features and peculiarities which need to be taken into consideration when performing valuations.

These characteristics can be related, on one hand, to their common structural features and, on the other hand, to technical peculiarities.

Structural features refer only to the enterprises that present the relevant qualitative characteristics. They refer essentially to the governance and to the socio-economic role played by the entity in the community where it operates.

Technical peculiarities are related to the usual practices that lead practitioners to consider an SME based on the assumption that it is a private company and usually less regulated.

Governance

One SMEs' common characteristic is that managers are often not substantially independent from owners. Specifically, owners may be the managers (FEE, 2001). This aspect needs to be carefully taken into account in the valuation from different points of view.

First, a smaller enterprise embodies an intangible value, that is sometimes difficult to measure. IDW provides some potential personal characterizations of smaller entities, strictly related to the characterizing activities of owners, such as the provision of services that are crucial for customer satisfaction (e.g., a professional or an expert whose know-how is key for the development of new products (IDW, 2014)).

The question arising from the circumstances described above is: will the entity be able to maintain the former potential after the possible withdrawal of its owner? SMEs' intangible capital could be high, non-represented in bookkeeping and difficult to measure. The case of a smaller entity, whose appreciation in the market relates to the credibility acquired by its director/owner, is quite common. What about the measurement of the value of the enterprise, once the director/owner will not be involved anymore?

This can contribute to address special hypotheses in valuing businesses, considering the capacity of the enterprise to produce the same level of earnings in the short and medium-term future.

The wealth of an enterprise could depend upon the capacity of managers. This aspect can be differently judged in the application of different valuation methods, where the operation excludes the continuity in the management team. In this case, if the practitioner is going to use, for example, the market value, the valuation has to reflect the estimated amount for which the entity should be exchanged "between a willing buyer and a willing seller in an arm's length transaction, after proper marketing and where the parties had each acted knowledgeably, prudently and without compulsion" (IVSC, 2017), but without the added value that could be provided by the owners in their quality of managers. At the same time, the entity may benefit from special synergies arising from the personal owners' activities or assets. This can be the case of a synergistic complex built aside the entity, that is not part of the operation, or the case of a machinery, whose productivity is related to specific knowledge.

In addition, it is not rare that owners obtain, in their capacity of management, a higher (or lower) wage for their job, and such remuneration may sometimes not be related to the real "contribution" provided to the enterprise. Even this aspect should be appropriately considered when determining the flows of the enterprise. Their salary could be then compensated by the adopted dividend distribution policy (Pratt *et al.*, 1996). The perspective of the acquirer is clearly different and must preliminarily understand the related ongoing structural assumptions.

As an alternative to higher wages, owners can benefit from the distribution of resources based on specific profits distribution policies. Considering the above, it may conversely happen that the owner decides to distribute to himself less than the average, as they consider the entity as family. In substance, the policy in distributing profits can diverge from market conditions and impact the valuation.

An intuitive (and not easy) solution would be to isolate the enterprise from the abnormal (positive and negative) effects related to the observed facts and circumstances. The board of directors could be excessively numerous, as composed of several family members. The preliminary activity of the professional would be to value the enterprise in presence of a board of directors, based on the actual complexity and needs of the entity, as the change of the ownership would naturally bring the enterprise back to a physiological governance.

Corporate and personal assets

Owners/managers could, then, be tempted to combine personal and corporate affairs. The distinction between stockholders' and managers' interests, which is at the basis of the Agency theory (Jensen and Meckling, 1976; Eisenhardt, 1989), is blurred by the case of SMEs combining these two roles into one person (or one family).

In general terms, the practitioner should try to adequately distinguish corporate assets from "personal" assets. These last ones should/may not be related to the commercial side of the enterprise and be addressed as surplus assets.

In this context, even the relationship between the business and the owners has to be carefully analysed. IDW affirms that SMEs often do not have appropriate equity (IDW, 2008). That said, even the financial relationships between the ownership and the enterprise should be carefully examined. From the perspective of the enterprise, the separation between equity and liabilities is crucial (IDW, 2014). It can happen, for instance, that the owners fund the organization atypically. In addition, and in connection to this, SMEs often have a low level of equity, as the entrepreneur can be ready to invest only when necessary (or when they have the capacity to). This misleads the interpretation of financial statements, which might be altered by a contingent situation and could vary in relation to ongoing facts and circumstances. In this case, it would be relevant to assess whether the business debts are secured by the owner, as this could alter the deal with a potential acquirer. At the same time, specific attention should be dedicated to the items used promiscuously. The valuation should take into consideration the potential optimization of these elements; the valuation of an item used in part for a personal purpose and in part for business affairs could, for example, optimize its value for sale purposes.

Another issue relates to the strategy horizons. The non-formalized strategy of SMEs -especially when the enterprise is a family business- is often a long- term strategy. The perspective of the market might be shorter. Obviously, the strategy would change the composition of the estimated cash flows as well.

Tax systems

The corporate income tax of SMEs can be optimized with the owners' personal fiscal position. The tax effect should be referred to the enterprise, assumed as a stand-alone entity.

4.2. Technical peculiarities

General aspects

A preliminary critical feature when referring technical standards to the SMEs' environment concerns the collection of the required information and material.

In some cases, professional standards require collecting disclosures and information. This can worry practitioners, especially where information is not easy to obtain, due to the fact that SMEs are not required to make their disclosure public as they do not have a broad range of stakeholders. At the same time, due to their private position and to the lack of resources, SMEs do not present sophisticated organizations. The lack of reliable information, mainly in forward looking estimates, can create a significant issue. This could be the case of a practitioner who looks for structured strategy or innovative business models. Another common example is the absence of a structured plan, to consider the cash flows or benefits arising from the future operations. The technical and professional standards deal with this problem, often requiring the practitioner to comply with this duty, especially when this is essential to perform the valuation. In this case, the mentioned professional standards would allow practitioners to adopt the hypotheses and the assumptions of the directors, moving from historical data (IDW, 2014; OIV, 2015).

In addition, practitioners should establish whether -considering the lack of information- it is feasible for them to carry out the valuation engagement and should define the responsibilities in relation to the documentation provided.

Financial data and financial statements

Another important aspect to be focused concerns the reliability of the financial statements. In many jurisdictions, SMEs are not required to prepare general purpose financial statements. This implies that, at least in these circumstances, enterprises apply tax-based requirements to present their "true and fair view".

The scarce reliability of financial data can have a misleading impact. The appropriate determination of financial data is the basis of any business valuation. It is obvious that a misleading effect is produced especially when adopting the "accounting methods" (Penman, 2010). This is especially true when the valuation is carried out to reflect the point of view of an investor, who is willing to understand the earning power of the entity (Trugman, 2017). In this case, the asymmetry of information from "internal" operators and third parties can determine unreliable estimates, if the data are not accurate.

Additionally, many regulations allow SMEs to prepare their financial statements on an abbreviated basis. This reduces the ability of financial statements to provide an exhaustive picture of the entity's financial health.

Perception of risk

The management's impact and - even - perception of risk is usually different in SMEs and in large companies. SMEs are mostly mono-business. SMEs are, sometimes, related to stronger partners, specifically, clients. The concentration of business and/or clients implies in these cases the existence of higher risks in the management of the entity. At the same time, the lack of appropriate hedging policies can increase the possibility of a sudden financial crisis, which may threaten the going concern. However, it is true that the minor complexity of business models and financial data result in simpler plans and reduced uncertainty in the determination of forward data.

Illiquidity of entities

The illiquidity of smaller undertakings could create significant risks of marketability. The practitioner usually applies a discount to the determination of the value of an SME to reflect the potential difficulties that a vendor may find to sell an entity (Tuller, 2008; IVS, 2017) or qualified interests (IASB, 2013), particularly when the practitioner is measuring a fair value. However, some authors and organizations (IASB, 2013; Trugman, 2017) have proved that the degree of risk decreases when the size of enterprise increases (size premium). This generally requires the use of specific risk premiums for investing in SMEs as well (OIV, 2015; IRS, 2009). The measurement of the premium adjustments is always discretional and, according to this, questionable.

5. Potential criticalities regarding the use of approaches

5.1. Market approach

International and local practices are converging towards generally recognized standards. IVSC recognizes the following approaches: market approach; income approach; cost approach (IVSC, 2017). The following part of this paper investigates the adoption of the valuation methods belonging to the quoted approaches, often directly or indirectly promoted at a local professional level as well.

The adoption of the comparable approach proposes some clear issues in the valuation of SMEs. IVSC states that "[t]he market approach provides an indication of value by comparing the asset with identical or comparable (that is similar) assets for which price information is available" (IVSC, 2017, IVS 105, 20.1). The relevant identified valuation methods are: Comparable Transactions Method; Guideline publicly-traded comparable method.

In general terms, it is difficult for SMEs to be eligible to apply this approach, as many smaller enterprises can be unique in the market (AECA, 2005; Aznar *et al.*, 2016). Even when this is not the case, the market valuation methods can be subject to natural restrictions, as comparable assets are not existing (Heaton, 1998). IVSC affirms, for instance, that the comparable transaction methods can have natural limitations -and this should imply the application of adjustments- when the transactions dealt with are not recent enough, the assets are traded in non-active markets, comparable assets have significant differences, information is not reliable. All the recalled conditions can normally occur in the valuation of an SME.

The guideline publicly-traded method is not easy to apply for SMEs as it uses information on publicly-traded comparables that is obviously not often present for private enterprises. Public companies have different business models, and available data can be compared, only if adjusted with considerations that may be excessively discretional. Even financial data are often determined on different bases; while public companies apply internationally general accepted standards (IFRSs or US GAAPs), SMEs usually adopt local GAAPs.

It is very important to lastly observe that Small and Medium Sized Practices, that are often the first professionals involved in the valuation of SMEs, may sometimes lack the resources to obtain useful information to apply comparable methods, due to specific constraints.

This does not mean that the market approach cannot be applied to the SMEs valuation, even if it is quite evident that the preliminary collection of appropriate comparables is, especially in this circumstance, crucial and has to be carefully contextualized.

5.2. Income approach

As of today, the income approach is probably the most used approach in valuing businesses. It "... provides an indication of value by converting future cash flow to a single current value. Under the income approach, the value of an asset is determined by reference to the value of income, cash flow or cost savings generated by the asset" (IVSC, 2017, IVS 105, 40.1).

Despite stating that there are many valuation methods referring to the income approach, IVSC explicitly examines only the DCF, which values businesses as the summation of the discounted net cash flows -available to owners (free cash flows to equity) in the equity side or to the enterprise (free cash flow to the enterprise) in the asset side- and the terminal value, appropriately discounted (Fernández, 2013).

Sometimes the corporate "personalization" creates inevitable problems in the data interpretation as well. This analysis originates from the investigation of previous financial reporting, meant as "the starting point for the projection of future developments and for undertaking plausibility considerations" (IDW, 2008). The determination of the steady state income requires a careful normalization of cash flows and earnings.

This issue, examined above, requires attention in the determination of estimated cash flows, distinguishing "operating" cash flows from personal cash flows.

The adoption of DCF for SMEs is differently considered by the literature. Some authors consider it as the most valuable approach or the most appropriate approach if referred to smaller businesses (Heaton, 1998; AECA, 2005).

A recognized issue in adopting the income approach for SMEs, specifically referring to the DCF valuation method, is the determination of the discount rate and its variables. This issue arises especially from the fact that SMEs do not often have business plans and from the determination of the discount rate.

As regards the preparation of business plans, reference should be made to the considerations expressed in the previous paragraph.

In relation to the determination of the discount rate, the main criticality is that the applied formulas are usually based on public data that are naturally referred to public companies (Cheung, 1999). The adoption of the asset valuation perspective usually applies the WACC (Weighted Average Cost of Capital) as the discount rate.

$$\text{WACC} = K_e \times \frac{E}{D+E} + K_d \times (1-t) \times \frac{D}{D+E}$$

K_e = cost of equity
E = equity
D = debt
K_d = cost of debt
t = income taxes

It is beyond the scope of this paper to investigate the determination of the above-mentioned formula, except considering whether it is functional to its adaptation to the SMEs.

Having said that, the formula can be created only where data are publicly available and consequently relies on information referred to the public companies. According to this, the collected values could need to be adjusted when applied to SMEs.

The specific risk of the investment in SMEs represented by the cost of capital is usually lower for a public company than for a private company. This is motivated by the evidence that an investment in an SME normally reflects higher risks, due to the exposure to a less organized structure, lack of data, the illiquidity of the enterprise and the concentration of risks (OIV, 2015).

As regards the organizational and governance peculiarities, reference should be made to the considerations expressed in the previous paragraph.

Also, the appropriate determination of the Capital Asset Pricing Model (CAPM), necessary to estimate the discount rate under a levered and unlevered side, can be controversial. CAPM appeared to be applied by 90% of professionals using the DCF in their business valuations. It measures the minimum return that an equity investor can accept to enter the operation, that is the cost of equity.

The generally recognized formula states that the cost of equity equals the addition of a risk-free rate and the result of ß, as a measure of the volatility of the investment return in relation to the market as a whole (systematic risk), and the equity risk premium (excess return), given by the difference between the risk of return and the risk-free rate:

$$K_e = r_f + \beta \times (r_m - r_f)$$

R_f = free-risk rate
ß = beta
r_m = expected market return

The determination of the cost of SMEs' equity is a subject that has been studied for decades (Boyer and Roth, 1976). Some authors argue that the applied formula should be adjusted if referred to smaller enterprises in order to reflect some conditions that are not considered in the determination of the above-mentioned values.

Other authors and professional bodies have focused their attention on the determination of cash flows. AECA, the Spanish professional body, has addressed the lack of information, concentration of risk and illiquidity of the investment (AECA, 2005).

Beta assumes the existence of a list of peers in the market. The lack of a reliable peer group should require an adjustment to obtain a reliable systematic risk measurement. Specifically, beta measures the appreciation of risk for a diversified portfolio. The diversification naturally reduces the impact related to the performance of a specific company. Usually, the owners of SMEs do not diversify the risk and concentrate instead its capital on "their" operations. In many cases, minority interests do not exist. When this occurs, beta -meant as the market risk- provides an overrated measure of the ownership risk. In order to establish an appropriate level of risk, Damodaran has proposed to take into consideration the non-systematic risk, if the owners have not diversified their risk, by the determination of the "total beta" (Damodaran, 2002). This is computed as an adjustment to the original beta.

Damodaran's total beta is computed as:

$$\text{Total Beta} = \frac{Market\ Beta}{\rho_{jm}}$$

ρ_{jm} = correlation between the firm's equity and the market index

Market beta = ρ_{jm} (σ_j / σ_m) equal to the product of (i) ρ_{jm}, and (ii) σ_j / σ_m as the relation between σ_j = standard deviation of the firm's equity return and σ_m = standard deviation of the equity market return.

The formula highlights that the lower the correlation, the higher the total beta.

Regardless of what the literature has developed by the total beta, the body literature is not unanimously in favour of the adoption of total beta and has evidenced that experts do not consider the adjustments to the market beta in the estimate of private companies (Petersen *et al.*, 2006; Kasper, 2013).

Moreover, even if it is difficult to generalize, the cost of debt is usually considered higher than in larger companies for the reasons mentioned above, and specifically for the embodied higher risks, and due to the smaller investments (Badertscher *et al.*, 2017). The adoption of total beta for the valuation of private companies has been widely debated and, as mentioned, the theoretical and empirical effects are not generally accepted (Von Helfenstein, 2009 and 2011; Kasper, 2013).

5.3. Cost approach

As explained by IVSC "The cost approach provides an indication of value using the economic principle that a buyer will pay no more for an asset than the cost to obtain an asset of equal utility, whether by purchase or by construction, unless undue time, inconvenience, risk or other factors are involved. The approach provides an indication of value by calculating the current replacement or reproduction cost of an asset and making deductions for physical deterioration and all other relevant forms of obsolescence" (IVSC, 2017, IVS 105, 60.1).

The generally applied valuation methods are:

- replacement cost method: a method that indicates value by calculating the cost of a similar asset offering equivalent utility,
- reproduction cost method: a method under the cost that indicates value by calculating the cost to recreating a replica of an asset, and
- summation method: a method that calculates the value of an asset by the addition of the separate values of its component parts (IVSC, 2017, IVS 105, 70.1).

The summation method is sometimes considered as an appropriate method for the valuations of SMEs (Liberatore, 2010; Behringer, 2012, as mentioned by Bensh *et al.*, 2013). The adoption of the summation method (otherwise named as net asset value) could be due to the inappropriateness of the adoption of the market and/or the income approaches.

The method can *de facto* be lacking from the perspective of the determination of the intangible capital. In order to limit this distortive effect, practitioners usually apply diversified methods to adjust the value of SMEs arising from a pure cost approach. The category has been formalized by the Union des Experts Comptables (UEC, 1961) and then reproduced in several methods. One of the most known method, which can mix the net assets valuation and the income perspective, aggregates the net assets value and the measurement of the goodwill (badwill):

$$W = K + a_{n\neg i} (R - Ki)$$

K = net assets value
$a_{n\neg i}$ (R - Ki) = goodwill, where, in detail:
n = time horizon
i = cost of capital
R = normalized expected earnings

It is true that the methods that combine cost approaches and income approaches (Guatri and Bini, 2009) are progressively emphasizing the income component rather than the net assets component (OIV, 2015).

6. Conclusions and Recommendations

Based on the considerations presented in this paper, the first question we should address is whether *ad hoc* methods or standards for the valuation of SMEs are actually needed. The different sources of literature have discussed how to apply the valuation methods to SMEs, and sometimes determined adjustments to be used in the application of the original models but have proposed very few separate methods. Professional bodies have identified specific criteria within the standard models. The creation of different sets of valuation would likely damage the value of the valuation. It would probably result in the creation of two different categories with diverse levels of approach, which would appear quite an anomaly from a professional perspective.

Additionally, standards and best practices are usually recognized at a local level, either formally or informally. Each departure from the used approaches could create more difficulties than benefits. Academia often proposed, as mentioned above, appropriate deviations from the original models. Nevertheless, the proposals may sometimes bring to more relevant solutions from a purely theoretical point of view, which are however less feasible in practical terms.

The valuation of SMEs does not need *ad hoc* standards. Technical standards are often principle-based. In this case, standard setters, professional bodies and/or practitioners should aim to orient the standards in order to find practical solutions that ensure the application of best practices to the SMEs environment, as it has already happened.

The adaptation should allow to determine a scalable application, especially when the process is driven by public companies' experience. The "scalable approach" is already known, for instance, in the audit context, where practitioners and auditors have debated over the last years how the International Standards on Auditing should be applied to SMEs.

An adaptation of standards to the reality of SMEs and SMPs does not alter the rigour of a model but allows to apply the technical requirements in the circumstances of SMEs, which present -as observed in this paper- peculiarities and specific considerations.

It is evident that practitioners should always take into consideration the entity's specific characteristic to re-produce the required value firm. In conclusion, it is always the practitioner's task to "weigh up" the peculiarities of an SME. For instance, a non-large private company may have comparable entities in regulated markets. In this specific context, the market approach would be applicable. If a public company is an *unicum* in the market, the comparable valuation method is not applicable. To this purpose, the comprehension of the context is crucial to determine the value of an enterprise, irrespective of its size; and this approach is likely to be even more important in the valuation of smaller enterprises.

Lastly, it is important that SMEs are not considered as a marginal sector, as they are the crucial engine of local economies and their appropriate valuation needs to create an effective market where valuations play a significant role.

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